Lab worksheet

10/3/2017

1. A vector-valued function is also called a *vector field*. To plot a vector field, you can use the command *VectorPlot* or *VectorPlot3D*. Press F1 and type the keyword VectorPlot to learn the syntax.

   Now plot the vector fields $f_1(x, y) = \langle -y, x\rangle$, $f_2(x, y) = \langle x^2, xy\rangle$ and $f_3(x, y, z) = \langle \sin x, \sin y, \sin z\rangle$.

2. Let $f(x, y) = y^3 - x^3$. Plot the gradient vector field ∇f together with the level sets of f. Based on the plot, what can you tell?

3. (Exercises 1 and 3 in Lab 4) Let

$$T(x, y, z) = \frac{6000}{1 + \frac{1}{5}(x^2 + y^2 + z^2)}.$$

   (a) Calculate the directional derivative at point $A(9, 2, 1)$ in the direction of vector $\langle -4, -3, 5\rangle$.

   (b) At point A,

      (i) in what direction does the function have the largest increase?

(ii) in what direction does the function have the largest decrease?

(iii) in what direction does the function remain at the same value?

4. A vector-valued function that has single-valued variable is called a *curve*.

   Consider the curve $c_1(t) = \langle 8\sin t - \cos(2t), t - \sin t \rangle$.

   (a) Plot the curve.

   (b) Compute the tangent vector at point $t = 0$. Plot this tangent vector on the same graph with the curve. (You will need the command *Arrow* to draw a vector.)

   (c) Find all t where the tangent vector is parallel to the x-axis.

5. Consider the curve $c_2(t) = \langle t^3, t, t^2 \rangle$.

   (a) Plot the curve.

(b) Compute the tangent vector at point $t = 1$. Plot this tangent vector on the same graph with the curve.

(c) Find all t where the tangent vector is

(i) parallel to the plane $x + 6z = 0$

(ii) perpendicular to the plane $x + y = 1$

6. Let $f(x, y) = (2y + 1, e^{2x-4+y^2})$.

(a) Find the derivative (Jacobian) matrix of f. Double check your result with Mathematica by using the command $D[f[x, y], \{\{x, y\}\}]$.

(b) Find the Jacobian matrix at point (2,0).

(c) Give a linear approximation near point (2,0).

(d) Estimate $f(1.9, 0.01)$. Use Mathematica the compute the exact value of $f(1.9, 0.01)$. Are they close?

7. Let $f(x, y, z) = \langle xy, y + z^2 \rangle$ and $g(u, v) = e^{u+v} \cos(u + v)$. Put $h = g \circ f$.

(a) Using the chain rule, compute $\nabla h(0, -1, 1)$. Double check your result with Mathematica.

(b) Using the chain rule, compute $\dfrac{\partial h}{\partial z}$. Double check your result with Mathematica.