Lab worksheet
9/25/2017

1. Write the equation of the plane that contains point $A(1, 2, 3)$ and perpendicular to vector $\vec{n} = \langle 2, -1, 1 \rangle$. Plot the plane.

2. Write the equation of the plane that contains point $A(1, 2, -1)$ and parallel to two vectors $\vec{a} = \langle 0, 2, 1 \rangle$ and $\vec{b} = \langle -2, 1, 2 \rangle$.

3. The *parametric equation* of a plane which contains point $A(x_0, y_0, z_0)$ and parallel to two vectors $\vec{a} = \langle a_1, a_2, a_3 \rangle$ and $\vec{b} = \langle b_1, b_2, b_3 \rangle$ is

$$\begin{cases} x = x_0 + ta_1 + sb_1 \\ y = y_0 + ta_2 + sb_2 \\ z = z_0 + ta_3 + sb_3 \end{cases}$$

   In short notations, $P(t, s) = A + t\vec{a} + s\vec{b}$.

   Now write the parametric equation of the plane in Problem 2. Plot the plane. (You will need the command *ParametricPlot3D*)

4. Given a single-variable function $y = f(x)$.

   (a) Write coordinates of a point A on the graph whose x-coordinate is x_0.

   (b) Write the equation of the line passing through A and tangent to the graph.

   (c) Write a direction vector of that line.

5. Given a two-variable function $z = f(x, y)$.

   (a) Write coordinates of a point A on the graph (surface) whose (x,y)-coordinates are (x_0, y_0).

   (b) The cross section $x = x_0$ of the surface is a curve whose equation is $z = f(x_0, y)$. Write a direction vector of the tangent line to this curve at point A.

   (c) The cross section $y = y_0$ of the surface is a curve whose equation is $z = f(x, y_0)$. Write a direction vector of the tangent line to this curve at point A.

   (d) Write a parametric equation of the plane tangent to the surface at A.

6. (Exercise 2 of Lab 3) Given the function $g(x, y) = x^2 - y^2$. Write a parametric equation for the tangent plane of fucntion g at (1,2). Plot the surface and together with the tangent plane. (You will need the command *Show*.)

7. Given a two-variable function $z = f(x, y)$. Define $g(x, y, z) = z - f(x, y)$.

   (a) What is the level set $g(x, y, z) = 0$ in relation to the graph of function f?

   (b) Using the principle "a level set is perpendicular to the gradient vector', determine a normal vector of this level set at point (x_0, y_0, z_0)?

   (c) What is the equation of the equation of the tangent plane of the graph of f at point $A(x_0, y_0, z_0)$?

8. (Exercise 4 of Lab 3) Given the function $f(x, y) = xe^{-x^2-y^2}$. Using the method in Problem 7, find the equation of the plane tangent to the graph of f at point $(0, 0, f(0, 0))$.