Lab worksheet

10/10/2017

There are several ways to visualize a 2-dimensional or 3-dimensional region in Mathematica. Some helpful commands are *Plot(3D)*, *ContourPlot(3D)*, and *RegionPlot(3D)*. Let's first consider how to sketch a 2-dimensional region (Problem 1 and 2).

1. Visualize $R = \{(x, y) : a \leq x \leq b,\ f(x) \leq y \leq g(x)\}$:

   Plot[{f(x), g(x)}, {x, a, b}, Filling→{1→{2}}, PlotLegends→"Expressions"]

   The part "1→{2}" means to fill the region from the graph of f(x) (the *first* function) to the graph of g(x) (the *second* function). The "PlotLegends" part is optional, specifying on the plot which curve is of which function.

   Practice

   (a) Visualize $R = \{(x, y) : 0 \leq x \leq 1,\ x^2 \leq y \leq x\}$.

   (b) Visualize $R = \{(x, y) : 0 \leq x \leq 1,\ 0 \leq y \leq 1\}$.

   (c) Visualize $R = \{(x, y) : 0 \leq y \leq 1,\ 0 \leq x \leq 2 - y\}$.

2. When R is described not as explicitly as in Part 1, we can use the command *RegionPlot*. For example, if R is a domain inside the circle $x^2 + y^2 = 9$ and outside the circle $(x-1)^2+(y+1)^2 = 1$, we can write R as $R = \{(x, y) : x^2+y^2 = 9,\ (x-1)^2+(y+1)^2 = 1\}$. Command:

   RegionPlot[$x^2 + y^2 \leq 9$ && $(x - 1)^2 + (y + 1)^2 \geq 1$, {x, -3, 3}, {y, -3, 3}]

   Practice

   (a) Sketch the region in the first quadrant, under the line $y = 2 - x$.

(b) Sketch the region inside the ellipse $\frac{x^2}{4} + (y-2)^2 = 1$, on the right of the line $x = 1$.

For visualizing a 3-dimensional region, the color effects become more important. Let's consider some examples (Problem 3 and 4).

3. Visualize $R = \{(x, y, z) : a \le x \le b,\ f(x) \le y \le g(x),\ h(x, y) \le z \le k(x, y)\}$:

   RegionPlot3D[$a \le x \le b$ && $f(x) \le y \le g(x)$ && $h(x, y) \le z \le k(x, y)$,
   PlotStyle→Opacity[0.5], Mesh→None]

   Practice

   (a) Sketch the region in the first octant (meaning x, y, z are nonnegative), under the paraboloid $z = 1 - x^2 - y^2$.

   (b) Sketch the region inside the ellipsoid $\frac{x^2}{4} + y^2 + z^2 = 1$, above the plane $x + y + z = 0$.

   (c) Sketch the region for the integral

   $$\int_0^1 \int_0^{2x} \int_{x^2+y^2}^{x+y} f(x, y, z)\,dz\,dy\,dx$$

4. Sometimes a 3D region is best visualized by plotting all surfaces (that form it) at once. For example, when R is the region cut out of the ball $x^2 + y^2 + z^2 \le 4$ by the cylinder $x^2 + z^2 = 1$:

ContourPlot3D[{$x^2 + y^2 + z^2 == 4$, $2x^2 + z^2 == 1$}, {x,-2,2}, {y,-2,2}, {z, -2, 2}, ContourStyle→Opacity[0.5], Mesh→None]

Practice

(a) Visualize the region between the cone $z = \sqrt{x^2 + y^2}$ and the paraboloid $z = x^2 + y^2$.

(b) Visualize the region bounded by $z = 0$, $z = \pi$, $y = 0$, $y = 1$, $x = 0$ and $x + y = 1$.

There are several ways to compute a double/triple integral in Mathematica (Problem 5 and 6).

5. One way is to use do an iterated integral, which involves writing the domain in such a way that we can take integral with respect to each variable, one by one. For example, to compute the integral given by Part (c) of Problem 3, we execute

Integrate[f(x, y, z),{x, 0, 1}, {y, 0, 2x} ,{z, $x^2 + y^2$, x+y}]

Practice

(a) Compute

$$\int_0^1 \int_{-x}^{x} \int_{-x+y}^{x+y} (x^2 + zy)dzdydx$$

(b) Compute the volume of the region in Part (a) of Problem 4.

6. Another way is to *define the region of integration*. Then let Mathematica compute the double/triple integral. For example, the region the region R at the beginning of Problem 3 can be defined in Mathematica as

   R = ImplicitRegion[$a \leq x \leq b$ && $f(x) \leq y \leq g(x)$ && $h(x,y) \leq z \leq k(x,y)$, {x, y, z}]

   Then the integral

   $$\iiint\limits_R u(x,y,z)dV$$

   is evaluated via the command

   Integrate[u(x, y, z), {x, y, z}∈R]

   The symbol "∈" can be typed as ESC el ESC.

   Practice

   (a) Compute the volume of the region in Part (b) of Problem 3.

   (b) Compute the area of the region in Part (b) of Problem 2.