Lab worksheet

10/24/2017

1. There are several ways to plot a surface over a region (instead of a rectangle as we usually do). One way is to use Plot3D with some modification in the range. For example, to plot the surface $z = xy^3$ over the circle $x^2 + y^2 = 1$, we write

$$\mathrm{Plot3D[xy^3,\ \{x, 0, 1\},\ \{y, -\sqrt{1-x^2}, \sqrt{1-x^2}\}]}$$

   Another way is to use the option *RegionFunction* inside Plot3D. For example, the above graph can be plotted by

$$\mathrm{Plot3D[xy^3,\ \{x, 0, 1\},\ \{y, 0, 1\},\ \ RegionFunction \rightarrow Function[\{x, y\},\ x^2 + y^2 \leq 1]}$$

   If you add the option $Filling \rightarrow Bottom$, it will even look nicer.

   Practice

   (a) Plot the surface $z = xy^2$ over the annulus centered at (0,0) with inner radius 1 and outer radius 2.

   (b) Plot the same function over the ellipse $(x - 1)^2 + 4(y - 1)^2 = 9$.

2. To compute gradient, divergence, and curl of a vector field, use the commands *Grad*, *Div* and *Curl* respectively. For example,
   Grad[F(x,y,z),{x,y,z}]
   Div[F(x,y),{x,y}]
   Curl[F(x,y,z),{x,y,z}]
   ...
   A remark before practice: usually we only define curl for a 3-variable vector field. However, given a 2-variable vector field, we still have a way define the curl. For example, let $F(x, y) = \langle f(x, y), g(x, y)\rangle$. This function can be thought as $\tilde{F}(x, y, z) = \langle f(x, y), g(x, y), 0\rangle$, whose curl is

$$\left\langle 0, 0, \frac{\partial g}{\partial x} - \frac{\partial f}{\partial y} \right\rangle$$

   Thus, the curl of F can be defined as

$$\text{curl } F = \frac{\partial g}{\partial x} - \frac{\partial f}{\partial y},$$

   which is a scalar.

Practice

(a) Compute the divergence and gradient of $F(x,y,z) = \langle xy^2, yz^2, zx^2\rangle$.

(b) Let $F(x,y) = \langle x, y\rangle$. Plot vector field F. (*You will need the VectorPlot command. Press F1 to look up the syntax if you forget*). Explain intuitively based on the plot why the divergence at (0,0) and at (1,1) must be positive.

(c) Let $F(x,y) = \langle -y, x\rangle$. Plot vector field F. Explain intuitively based on the plot why the divergence at (1,1), and why curl at (0,0) must be positive.

(d) Let $F(x,y) = \langle -x, -x\rangle$. Plot vector field F. Explain intuitively based on the plot why the curl at (1,1) must be negative.

(e) Let $f(x,y) = xy^2$. Plot the gradient vector field of f. Explain intuitively based on the plot why the curl of the gradient must be zero.