Worksheets
10/19/2017

1. Compute the length of the cycloid $c(t) = (t - \sin t, 1 - \cos t)$, $t \in [0, 2\pi]$.
   [*Hint: You will need the trigonometric identity* $1 - \cos(2\theta) = 2\sin^2\theta$.]

2. Let $F(x, y, z) = \langle xy^2, y + zx, zx^2 \rangle$. Find div F, curl F and div ∇F.

3. A wire is parametrized by $c(t) = (\cos t, \sin t, t)$, $0 \leq t \leq \frac{\pi}{2}$. Let its mass density at point (x, y, z) be given by $f(x, y, z) = xy$. Find the mass of the wire.

   [*Hint: You will need the trigonometric identity* $\sin(2\theta) = 2\sin\theta\cos\theta$.]

4. (Exercise 25, p.259) Let $F : \mathbb{R}^3 \to \mathbb{R}^3$ be a vector field. Which of the following expressions are meaningful, and which are nonsense? For those which are meaningful, decide whether the expression defines a scalar function or a vector field.

   (a) curl(grad F)

   (b) grad(curl F)

   (c) div(grad F)

   (d) grad(div F)

   (e) curl(div F)

   (f) div(curl F)

5. Find the work done by the force field $F(x, y) = \langle x, 1 \rangle$ on a particle that moves along the cycloid in Problem 1.

6. Fill in the blanks

$$\int_0^1 \int_x^1 \int_0^{y-x} f(x, y, z) dz dy dx = \int \int \int \qquad dx dy dz$$

Then sketch the region for the integral.

7. Find a parametrization for the curve C which is the intersection of

   (a) the cylinder $y^2 + z^2 = 1$ and the plane $z = x$

   (b) the cone $z = \sqrt{x^2 + y^2}$ and the sphere $x^2 + y^2 + z^2 = 2$.

8. Compute the volume of the solid bounded by the planes $x = 0$, $y = 0$, $z = 0$, $x + y = 0$ and $x = z - y - 1$.