Worksheets

10/26/2017

1. Evaluate the following line integrals $\int_C f(x, y, z)ds$, where

   (a) $f(x, y, z) = x + y + z$ and $C(t) = (\sin t, \cos t, t)$, $t \in [0, 2\pi]$

   (b) $f(x, y, z) = yz$ and $C(t) = (t, 3t, 2t)$, $t \in [1, 3]$

2. Find an parametrization of the ellipse

$$\frac{(x-2)^2}{4} + \frac{(y-3)^2}{9} = 1$$

oriented counterclockwise. Then express the length of the ellipse as an integral (but don't evaluate).

3. Evaluate the following line integrals:

   (a) $\int_C x dy - y dx$ where $C(t) = (\cos t, \sin t)$, $t \in [0, 2\pi]$

(b) $\int_C dy + dx$ where C is the ellipse given in Problem 2.

4. Consider the force field $F(x, y, z) = \langle x, y, z \rangle$. Compute the work done in moving a particle along the parabola $y = x^2$, $z = 0$ from $x = -1$ to $x = 2$.

5. A wire has parametrization $c(t) = (t^2, t, 3)$, $0 \leq t \leq 1$. The mass density at point (x, y, z) is given by $\rho(x, y, z) = y$. Compute the mass of the wire.

Recall the Green's theorem

$$\int_{\partial D} Pdx + Qdy = \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dA$$

where ∂D denotes the boundary of D, positively oriented.

6. Let D be a region on the plane. Choose appropriate P and Q so that

$$\text{Area}(D) = \iint_D 1 dA = \int_{\partial D} Pdx + Qdy$$

7. Use the formula in Problem 6 to compute the area of the ellipse in Problem 2.