## Worksheets

10/5/2017

1. Compute the integral

$$
\iint_{D} x(x+y) d A
$$

where $D$ is the rectangle defined by $-1 \leq x \leq 1$ and $0 \leq y \leq 1$.
2. Fill in the following table

| Domain $D$ | $\iint_{D} u(x, y) d A=$ |
| :---: | :---: |
| $a \leq x \leq b$ and $c \leq y \leq d$ |  |
| $a \leq x \leq b$ and $f(x) \leq y \leq g(x)$ |  |
| $a \leq y \leq b$ and $f(y) \leq x \leq g(y)$ |  |

3. For each of the domain $D$ below, express it in form of one of the 3 types in the previous problem.
(a) The rectangle with vertices $(0,0),(2,0),(2,1),(0,1)$.
(b) The triangle with vertices $(0,0),(2,0),(2,1)$.
(c) The triangle with vertices $(0,0),(2,1),(2,-1)$.
(d) The triangle with vertices $(0,0),(1,2),(2,0)$.
4. Compute the integral

$$
\iint_{D} x(x+y) d A
$$

where $D$ is the triangle with vertices $(0,0),(2,0)$ and $(2,1)$.
5. You have learned (or will learn) an important property of integration: Suppose the domain $D$ can be "split" into two non-overlapping domains $D_{1}$ and $D_{2}$. Then

$$
\iint_{D} u(x, y) d A=\iint_{D_{1}} u(x, y) d A+\iint_{D_{2}} u(x, y) d A .
$$

An application is that: to find the integral of a function over a domain $D$ which is not of any types in Problem 2, we try to split $D$ into subdomains so that each belongs to one of the types in Problem 2. The integration on each subdomain can be computed. Then we add them together.

Let's practice! Split the following domains into two (or more) subdomains, each of which is of one of the types in Problem 2.
(a) $D$ is the triangle with vertices $(1,1),(2,3),(4,2)$.
(b) $D$ is the quadrilateral with vertices $(0,0),(1,1),(3,2),(4,0)$.

