Worksheets

11/16/2017

1. Consider a surface given by $x = 2u$, $y = u^2 + v$, $z = v^2$.

   (a) Find two tangent vectors to the surface at point A(0,1,1).

   (b) Find a normal vector to the surface at point A.

   (c) Write *cartesian* and *parametric* equations for the tangent plane at point A.

2. Give a parametrization for the following surfaces

   (a) The single cone $z^2 = x^2 + y^2$, $z \geq 0$

(b) The elliptic cone $z^2 = 4x^2 + 9y^2$, $z \geq 0$

(c) The unit sphere $x^2 + y^2 + z^2 = 1$

(d) The ellipsoid $x^2 + 4y^2 + \dfrac{(z-1)^2}{9} = 1$

3. Let S be the ellipsoid $x^2 + 4y^2 + \dfrac{(z-1)^2}{9} = 1$.

   (a) Apply the principle "Level sets of a function are perpendicular to its gradient vectors" to find a normal vector to S at point $\mathrm{A}\left(\frac{1}{3}, \frac{1}{3}, 3\right)$.

(b) Write the cartesian equation for the tangent plane at point A.

(c) Compared to Problem 1, what makes it (a little) easier for us to find the tangent plane in this problem.

4. Consider a surface S given by $x = u^2$, $y = v^2$, $z = uv$, $0 \leq u, v \leq 1$.

(a) Find the tangent vectors T_u and T_v

(b) Calculate $\|T_u \times T_v\|$

(c) Find the area of S

5. A surface S is given by parametric equations $x = u - v$, $y = u + v$, $z = uv$, where $u^2 + v^2 \leq 1$.

(a) Find the area of the surface

(b) Compute the integral

$$\iint_S (x + y) dS$$