Worksheets
11/2/2017

1. Evaluate the line integral $\int_c x^2 y\,dx + y\,dy$, where c is the boundary of the region between the curves $y = x$ and $y = x^3$, $0 \le x \le 1$, oriented counter-clockwise, by

    (a) Using the definition of line integral (of a vector field)

    (b) Using Green's theorem

2. Evaluate the circulation of $F(x,y) = \langle x + y,\ xy \rangle$ around the rectangle $0 \le x \le 2$, $0 \le y \le 3$, by

(a) Using the definition of line integral (calculate the line integral on each edge of the rectangle, then sum all up).

(b) Using Green's theorem

3. Recall that a vector field is called conservative if it is the gradient of a scalar function. If a vector field is defined in the whole plane or space, it is conservative if and only if its curl is equal to zero.

Decide if each of following vector field is conservative. If it is, find a function f such that $F = \nabla f$

(a) $F(x, y) = \langle y^2 e^{xy^2}, \ 2ye^{xy^2} \rangle$

(b) $F(x, y, z) = \langle 3x^2y, \ x, \ 5 \rangle$

(c) $F(x, y, z) = \langle x + z, \ -y - z, \ x - y \rangle$

4. (a) Compute the Jacobian determinant $\frac{\partial(x,y)}{\partial(r,\theta)}$ of the change of variables to polar coordinates $x = r\cos\theta, \ y = r\sin\theta$

(b) Let D be the unit disk $x^2 + y^2 \leq 1$. Evaluate

$$\iint_D \exp(x^2 + y^2)dxdy$$

by making the change of variables to polar coordinates.

5. Integrate $x^2 + y^2 + z^2$ over the cylinder $x^2 + y^2 \leq 2$, $-2 \leq z \leq 3$.