Worksheets 12/12/2017

1. Find and classify all critical points of the function

$$g(x,y) = \frac{1}{4}x^4 - \frac{5}{3}x^3 + y^3 + 3x^2 - \frac{3}{2}y^2 + 20$$

2. Find maximum and minimum values of $f(x,y) = x(x^2 + y^2 - 1)$ on the disk $x^2 + y^2 \le 1$.

3. Let
$$f(x, y) = \frac{1}{x^2 + y^2}$$

(a) Find the equation of a plane tangent to the graph z = f(x, y) at x = 0, y = 1, z = f(0, 1).

(b) Find linear approximation of f at (x, y) = (0, 1).

4. Let

$$f(x,y) = (x^2 - 3, \ 3xy - y^3), \quad g(u,v) = (2uv, \ u^2 - v^2, \ u + v).$$
 Compute $D(g \circ f)(1,2).$

5. Evaluate the integral

$$\int_0^2 \int_{2y}^4 8\sqrt{x^2 + 1} dx dy$$

by changing the order of integration.

6. Evaluate $\iiint_E z dV$ where V is the region between the cylinder $x^2 + y^2 = 1$ above the xy-plane and below the cone $z = \sqrt{x^2 + y^2}$.

7. Let S be the boundary of the tetrahedron with vertices at (0,0,0), (1,0,0), (0,1,0), (0,0,1). Use the Divergence Theorem to compute

$$\iint_{S} \vec{F} \cdot d\vec{S}$$

where $\vec{F}(x, y, z) = (xy, xz, y)$.