# Worksheets

12/7/2017

1. Let $\vec{F} : \mathbb{R}^3 \to \mathbb{R}^3$ be a vector field, and $c = c(t)$ be a curve. You have learned three methods to compute $\int_c \vec{F} \cdot d\vec{s}$.

   (a) What are they?

   (b) In what situations is one method more favorable than the others?

   (c) Use your judgment to choose a method to compute $\int_C \vec{F} \cdot d\vec{s}$ where

      i. $\vec{F}(x, y, z) = (x,\ y,\ z)$, and $c(t) = (t^2, 3t, 2t^3)$, $0 \leq t \leq 1$.

ii. $\vec{F}(x,y,z) = (yz,\ zx,\ xy)$, and $c$ consists of straight lines joining (1,0,0), (0,1,0), (0,0,1) (in this order).

2. Let $\vec{F} : \mathbb{R}^3 \to \mathbb{R}^3$ be a vector field, and $S = \Phi(u,v)$ be a surface. You have learned two methods to compute $\int_S \vec{F} \cdot d\vec{S}$.

   (a) What are they?

   (b) In what situations is one method more favorable than the other?

(c) Use your judgment to choose a method to compute $\int_S \vec{F} \cdot d\vec{S}$ where

i. $\vec{F}(x, y, z) = (2x,\ -2y,\ z^2)$, and $S$ is the cylinder $x^2 + y^2 = 4$ with $0 \leq z \leq 1$.

ii. $\vec{F}(x, y, z) = (2x,\ y^2,\ z^2)$, and $S$ is the unit sphere $x^2 + y^2 + z^2 = 1$.

3. Give a parametrization to the following shape (curve/ surface/ solid):

(a) The intersection of the cylinder $x^2 + (y-1)^2{=}1$ and the graph $z = x^2 - y^2$.

(b) The triangle with vertices (1,0,0), (0,2,0), (0,1,1).

(c) The upper half ellipsoid

$$E = \left\{ (x, y, z) : \frac{x^2}{4} + (y-1)^2 + 4z^2 = 1, \quad z \geq 0 \right\}$$

4. Let $f : \mathbb{R}^2 \to \mathbb{R}$. Denote $\vec{X} = \begin{bmatrix} x \\ y \end{bmatrix}$ and $\vec{a} = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$. Recall that the *second-order* Taylor approximation of $f$ near $\vec{a}$ is

$$f(\vec{X}) \approx f(\vec{a}) + \nabla f(\vec{a})(\vec{X} - \vec{a}) + \frac{1}{2}(\vec{X} - \vec{a})^T \; Hf(\vec{a}) \; (\vec{X} - \vec{a})$$

Here $Hf(\vec{a})$ is the Hessian matrix.
Now compute the second-order Taylor approximation for the function $f(x, y) = e^x \cos y$ near the point $\vec{a} = (0, 0)$. Then find an approximate value of $f$ at (.1, -.01).