Worksheets
9/28/2017

1. Given a function $f(x, y) = e^{x+y}\cos(xy)$,

   (i) Compute the gradient vector ∇f.

   (ii) Calculate the directional derivative $D_{\vec{a}}f$ at point (0,0) in the direction of vector $\vec{a} = \langle 2, -1\rangle$.

   (iii) Calculate the directional derivative $D_{\vec{b}}f$ at point (0,0) in the direction of vector $\vec{b} = \nabla f(0, 0)$.

2. Given a function $f : \mathbb{R}^2 \to \mathbb{R}^3$, $f(x, y) = (xy, x + y^2, \sin y)$,

(i) Compute the derivative matrix (or Jacobian matrix) Df.

(ii) Find the derivative matrix of f at point (1,0).

(iii) Calculate the linear approximation $L(x, y)$ of f at point (1,0).

(iv) Use the linear approximation above to estimate $f(1, -0.1)$.

3. Given a function $z = f(x, y) = x^2 - xy + y^2$ and a point $A(1, -1, 3)$ which lies on its graph.

   (a) The cross section $x = 1$ of the graph is a curve. Write a direction vector of the tangent line to this curve at point A.

   (b) The cross section $y = -1$ of the graph is a curve. Write a direction vector of the tangent line to this curve at point A.

   (c) Write a parametric equation of the plane tangent to the graph at A.

4. Let f be the same function as in Problem 3. Define $g(x, y, z) = z - f(x, y)$.

   (a) What is the level set $g(x, y, z) = 0$ in relation to the graph of function f?

   (b) Using the principle "a level set is perpendicular to the gradient vector", determine a normal vector of this level set at point $A(1, -1, 3)$.

   (c) What is the (cartesian) equation of the tangent plane of the graph of f at point A?

5. Let $f : \mathbb{R}^2 \to \mathbb{R}$. Put $x = ts$, $y = t + s$ and $g(t, s) = f(x, y) = f(ts, t + s)$. Express the

$$\frac{\partial g}{\partial t}, \ \frac{\partial g}{\partial s}, \ \frac{\partial^2 g}{\partial t \partial s}$$

in terms t, s and partial derivatives of f.