6511 Review of Finding Antiderivatives

Find each of the following antiderivatives. The problems are easier if you use a u-substitution.

1. $\int e^{\cos(3x)} \sin(3x)\,dx$

Let $u = \cos(3x)$, then $du = -3\sin(3x)\,dx$

$-\frac{1}{3}\int e^u\,du = -\frac{1}{3}e^u + C = -\frac{1}{3}e^{\cos(3x)} + C$

2. $\int (\sin 3x)^4 \cos 3x\,dx$

Let $u = \sin(3x)$, then $du = 3\cos(3x)\,dx$

$\frac{1}{3}\int u^4\,du = \frac{1}{15}u^5 + C = \frac{1}{15}[\sin(3x)]^5 + C$

3. $\int [\tan(5x)]^4 [\sec(5x)]^2\,dx$

Let $u = \tan(5x)$, then $du = 5\sec^2(5x)\,dx$

$\frac{1}{5}\int u^4\,du = \frac{1}{25}u^5 + C = \frac{1}{25}[\tan(5x)]^5 + C$

4. $\int \frac{x+2}{3x^2 + 12x + 14}\,dx$

Let $u = 3x^2 + 12x + 14$, then $du = 6(x+2)\,dx$

$\frac{1}{6}\int \frac{du}{u} = \frac{1}{6}\ln|u| + C = \frac{1}{6}\ln|3x^2 + 12x + 14| + C$

5. $\int [4 + \cos(6x)]^{3/2} \sin(6x)\,dx$

Let $u = 4 + \cos(6x)$, then $du = -6\sin(6x)\,dx$

$-\frac{1}{6}\int u^{3/2}\,du = -\frac{1}{6}\cdot\frac{2}{5}u^{5/2} + C = -\frac{1}{15}[4 + \cos(6x)]^{5/2} + C$

Find each of the following antiderivatives. The problems are easier if you use a u-substitution.

1. $\int \frac{dx}{16 + 25x^2}$ Let $x = \frac{4}{5}u$, then $dx = (4/5)du$

$\frac{4}{5} \cdot \frac{1}{16} \int \frac{du}{1 + u^2} = \frac{1}{20} \arctan(u) + c = \frac{1}{20} \arctan\left(\frac{5x}{4}\right) + c$

2. $\int \frac{dx}{\sqrt{25 - 16x^2}}$ Let $x = \frac{5}{4}u$, then $dx = (5/4)du$

$\frac{5}{4} \cdot \frac{1}{5} \int \frac{du}{\sqrt{1 - u^2}} = \frac{1}{4} \arcsin(u) + c = \frac{1}{4} \arcsin\left(\frac{4x}{5}\right) + c$

3. $\int x(10 + 6x)^{2/3} dx$ Let $u = 10 + 6x$, then $du = 6dx$

$\frac{1}{36} \int u^{5/3} - 10u^{2/3} du = \frac{1}{96} u^{8/3} - \frac{1}{6} u^{5/3} + c$

$= \frac{1}{96}[10 + 6x]^{8/3} - \frac{1}{6}[10 + 6x]^{5/3} + c$

4. $\int \frac{x + 8}{x^2 + 1} dx = \int \frac{x}{x^2 + 1} dx + \int \frac{8}{x^2 + 1} dx$

$= \frac{1}{2} \ln(x^2 + 1) + 8 \arctan(x) + c$

5. $\int \frac{\cos(3x)}{[6 + 4\sin(3x)]^{3/2}}$ Let $u = 6 + 4\sin(3x)$ $du = 12\cos(3x)dx$

$\frac{1}{12} \int \frac{du}{u^{3/2}} = -\frac{1}{18} u^{-1/2} + c = \frac{-1}{18\sqrt{6 + 4\sin(3x)}} + c$

Note $6 + 4\sin(3x) > 0$