3513 Integration by Parts

The integration by parts formula is $\int f(x)g'(x)dx = f(x)g(x) - \int f'(x)g(x)dx$.

1. Evaluate the integral $\int x^4(\ln x)dx$.

Let $f(x) = \ln x$ And $g'(x) = x^4$, then $f'(x) = \frac{1}{x}$ $g(x) = \frac{x^5}{5}$

$$\int x^4(\ln x)\,dx = \frac{x^5}{5}\ln x - \int \frac{1}{x}\frac{x^5}{5}\,dx$$

$$= \frac{x^5}{5}\ln x - \frac{x^5}{25} + C$$

2. Evaluate the integral $\int x^2 \cos 5x dx$.

Let $f(x) = x^2$ $g'(x) = \cos(5x)$, then $f'(x) = 2x$ $g(x) = \frac{1}{5}\sin(5x$

$$= \frac{x^2}{5}\sin(5x) - \frac{2}{5}\int x \sin(5x)\,dx$$

$$= \frac{x^2}{5}\sin(5x) - \frac{2}{5}\left[-\frac{x}{5}\cos(5x) + \int \frac{1}{5}\cos(5x)\,dx\right]$$

$$= \frac{x^2}{5}\sin(5x) + \frac{2x}{25}\cos(5x) - \frac{2}{125}\sin(5x) + C$$

3. Given $f(1) = 3$, $f(5) = 10$, $f'(1) = 4$, $f'(5) = 7$, $\int_1^5 f(x)dx = 15$, $\int_1^5 f'(x)dx = 20$, $\int_1^5 x^2 f(x)dx = 25$, find the value of the integral

$$\int_1^5 xf'(x)dx.$$

$$\int_1^5 x f'(x)\,dx = x f(x)\Big|_1^5 - \int_1^5 1\, f(x)\,dx$$

$$= 5f(5) - f(1) - \int_1^5 f(x)\,dx = 32$$

6514 Integration using u-substitution

1. Find the antiderivative $\int \frac{x^3\,dx}{25+16x^2}$

Let $u = 25+16x^2$, then $du = 32x\,dx$ $\quad x\,dx = \frac{du}{32}$

$$\frac{1}{512}\int (1-25u^{-1})\,du = \frac{1}{512}\left[u - 25\ln|u|\right]+c$$

$$= \frac{1}{512}\left[(25+16x^2) - 25\ln(25+16x^2)\right]+c$$

2. Evaluate the indefinite integral $\int \frac{x^3\,dx}{\sqrt{25-16x^2}}$

Let $u^2 = 25-16x^2$ $\quad 2u\,du = -32x\,dx$ $\quad x\,dx = -\frac{1}{16}u\,du$

$$-\frac{1}{256}\int (25-u^2)\,du = -\frac{1}{256}\left[25u - \frac{u^3}{3}\right]+c$$

$$= \frac{1}{768}\left[25-16x^2\right]^{3/2} - \frac{25}{256}\left[25-16x^2\right]^{1/2}+c$$

3. Find the antiderivative $\int [8+3\sin(5x)]^{-1/2}(\cos 5x)\,dx$

Let $u = 8+3\sin(5x)$ $\quad du = 15\cos(5x)\,dx$

$$\frac{1}{15}\int u^{-1/2}\,du = \frac{2}{15}u^{1/2}+c$$

$$= \frac{2}{15}\left[8+3\sin(5x)\right]^{1/2}+c$$

4. Evaluate the indefinite integral $\int \frac{x^2+2}{5x^3+30x+24}\,dx$

Let $u = 5x^3+30x+24$ $\quad du = (15x^2+30)\,dx$

$$\frac{du}{15} = (x^2+2)\,dx$$

$$\frac{1}{15}\int \frac{du}{u} = \frac{1}{15}\ln|u|+c$$

$$= \frac{1}{15}\ln|5x^3+30x+24|+c$$

6515 More u-substitution

1. Find the antiderivative $\int [\cos 5x + \sin 5x]^{5/3} [\cos 5x - \sin 5x] dx$

Let $u = \cos 5x + \sin 5x$, then $du = (-5\sin 5x + 5\cos 5x)dx$

$\frac{1}{5} \int u^{5/3} du = \frac{1}{5} \cdot \frac{3}{8} u^{8/3} + c = \frac{3}{40} u^{8/3} + c$

$= \frac{3}{40} \left[\cos(5x) + \sin(5x) \right]^{8/3} + C$

2. Evaluate the indefinite integral $\int x(12 + 5x)^{3/2} dx$.

Let $u = 12 + 5x$, then $du = 5dx$ And $x = \frac{u-12}{5}$

$\int \frac{u-12}{5} u^{3/2} \frac{du}{5} = \frac{1}{25} \int u^{5/2} - 12 u^{3/2} du =$

$\frac{1}{25} \left[\frac{2}{7} u^{7/2} - \frac{24}{5} u^{5/2} \right]$

$= \frac{2}{175} (12+5x)^{7/2} - \frac{24}{125} (12+5x)^{5/2} + c$

3. Find the antiderivative $\int \frac{x^3}{(10 + 3x^2)^{3/5}} dx$, using $u^5 = 10 + 3x^2$.

Let $u^5 = 10 + 3x^2$, then $5u^4 du = 6x dx$

$\int \frac{u^5 - 10}{3} u^{-3} \frac{5u^4}{6} du = \frac{5}{18} \int (u^6 - 10u) du$

$= \frac{5}{126} (10 + 3x^2)^{7/5} - \frac{25}{18} (10 + 3x^2)^{2/5} + C$

4. Use integration by parts to evaluate the indefinite integral $\int x\sqrt{2x + 3} dx$.

Let $f(x) = x$ And $g'(x) = (2x+3)^{1/2}$

$f'(x) = 1$ $\quad g(x) = \frac{1}{3} (2x+3)^{3/2}$

$\int x (2x+3)^{1/2} dx = \frac{x}{3} (2x+3)^{3/2} - \frac{1}{3} \int (2x+3)^{3/2} dx$

$= \frac{x}{3} (2x+3)^{3/2} - \frac{1}{15} (2x+3)^{5/2} + C$