6521 Integration Using Trigonometric Identities

Some well known trigonometric identities

$$\sin^2 x + \cos^2 x = 1 \qquad \sin 2x = 2\ \sin x \cos x \qquad 2\sin^2 x = 1 - \cos 2x$$
$$\tan^2 x + 1 = \sec^2 x \qquad \cos 2x = \cos^2 x - \sin^2 x \qquad 2\cos^2 x = 1 + \cos 2x$$

Use these identities to evaluate the following integrals.

1. $\int \sin^3(bx)dx.$

$$= \int (1 - \cos^2 bx)\sin(bx)\,dx$$

$$= \frac{1}{b}\left[-\cos(bx) + \frac{\cos^3(bx)}{3}\right] + C$$

$u = \cos(bx)$

2. $\int \sin^2(bx)dx.$

$$= \frac{1}{2}\int (1 - \cos(2bx))\,dx$$

$$= \frac{x}{2} - \frac{1}{4b}\sin(2bx) + C$$

3. $\int \sin^2(4x)\cos^3(4x)dx.$

$$= \frac{1}{4}\int \sin^2(4x)\left[1 - \sin^2(4x)\right]\cos(4x)(4dx)$$

$$= \frac{\sin^3(4x)}{12} - \frac{\sin^5(4x)}{20} + C$$ using $u = \sin(4x)$

4. $\int \sin^2 x \cos^2 x dx.$

$$= \frac{1}{8}\int (1 - \cos(4x))\,dx$$

$$= \frac{x}{8} - \frac{1}{32}\sin(4x) + C$$

$$= \frac{x}{8} - \frac{1}{8}\sin x\cos^3 x + \frac{1}{8}\sin^3 x\cos x + C$$

6522 Partial Fractions

1. Find the antiderivative $\int \frac{18x+20}{9x^2+16}dx.$ $\qquad 3X=4u$

$$\int \frac{20dx}{9x^2+16} = \int \frac{20(4/3)du}{16u^2+16} = \frac{5}{3}\arctan u + C = \frac{5}{3}\arctan\left(\frac{3x}{4}\right)+C$$

$$\int \frac{18x\,dx}{9x^2+16} = \ln(9x^2+16)+C$$

$$\int \frac{18x+20}{9x^2+16}dx = \ln(9x^2+16) + \frac{5}{3}\arctan\left(\frac{3x}{4}\right)+C$$

2. Find constants A and B such that

$$\frac{A}{x-8}+\frac{B}{x+6}=\frac{2x+54}{(x-8)(x+6)}.$$

$A(x+6)+B(x-8) = 2v\cdot$

$(A+B)x + (6A-$

$A+B=2 \qquad 6A-$

$A=5 \qquad B=-$

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$$\frac{5}{x-8}-\frac{3}{x+6}$$

3. Find constants $A, B,$ and C such that

$$\frac{x^2-21x-107}{(x^2+9)(x-4)}=\frac{Ax+B}{x^2+9}+\frac{C}{x-4}.$$

$(Ax+B)(x-4)+C(x^2+9) = x^2-21x-107$

$(A+C)x^2+(-4A+B)x-4B+9C = x^2-21x-107$

$A+C=1 \qquad A=8$

$-4A+B=-21 \qquad B=11$

$-4B+9C=-107 \qquad C=-7$

$$\frac{8x+11}{x^2+9}-\frac{7}{x-4}$$