# 6523 Converting Definite Integrals

1. Convert the following definite integral into another definite integral using the substitution $x = (5/2)u$. Evaluate the resulting definite integral.

$$\int_{5/2}^{5\sqrt{3}/2} \frac{x^2\,dx}{4x^2+25}$$

$$\frac{5}{8}\int_1^{\sqrt{3}} \frac{u^2\,du}{1+u^2}$$

$$= \frac{5}{8}\int_1^{\sqrt{3}} \left(1 - \frac{1}{1+u^2}\right)du$$

$$= \frac{5}{8}\left[u - \arctan u \Big|_1^{\sqrt{3}}\right] = \frac{5}{8}\left[(\sqrt{3}-1) - \frac{\pi}{12}\right]$$

2. Convert the following definite integral into another definite integral using the substitution $x^2 = (16/9)(1-u^2)$ for $u > 0$.

$$\int_{2/3}^{2/\sqrt{3}} \frac{x^3\,dx}{\sqrt{16-9x^2}}$$

$$x^2 = \left(\frac{16}{9}\right)(1-u^2)$$

$$x\,dx = -\frac{16}{9}(u\,du)$$

$$u^2 = 1 - \frac{9}{16}x^2$$

when $x = 2/3$  $u = \frac{\sqrt{3}}{2}$

when $x = 2/\sqrt{3}$  $u = 1/2$

$$-\frac{64}{81}\int_{\sqrt{3}/2}^{1/2} (1-u^2)\,du$$

$$= \frac{64}{81}\left(\frac{\sqrt{3}}{2} - \frac{1}{2} - \frac{\sqrt{3}}{8} + \frac{1}{24}\right)$$

$$\frac{8\sqrt{3}}{27} - \frac{88}{243}$$

$$= 0.1511$$

3. Find an approximate value of the integral $\int_1^4 (4x - x^2)\,dx$ using the trapezoid rule with $n = 6$.

$$\frac{1/2}{2}\left[3+0+2\left(\frac{15}{4}+4+\frac{15}{4}+3+\frac{7}{4}\right)\right] = \frac{71}{8} = 8.875$$

4. Convert the following definite integral into another definite integral using the substitution $u^2 = 3x + 4$ with $u \geq 0$. Evaluate the resulting definite integral.

$$\int_7^{15} \frac{dx}{x\sqrt{3x+4}}$$

when $x = 7$  $u = 5$

when $x = 15$  $u = 7$

$$2\int_5^7 \frac{du}{u^2-4} = \frac{1}{2}\int_5^7 \left(\frac{1}{u-2} - \frac{1}{u+2}\right)du$$

$$= \frac{1}{2}\ln\left|\frac{u-2}{u+2}\right| \Big|_7^{15}$$

$$= \frac{1}{2}\ln\frac{117}{85}$$

$$= 0.1598$$

# 6524 Integration and Partial Fractions

1. Evaluate the indefinite integral

$$\int \left[\frac{4}{2x+3} - \frac{6}{3x+5}\right]dx \quad \int \frac{2dx}{(2x+3)(3x+5)}$$

$$\frac{A}{2x+3} + \frac{B}{3x+5} = \frac{2}{(2x+3)(3x+5)}$$

$A = 4 \quad B = -6$

$$= 2\ln|2x+3| - 2\ln|3x+5| + c$$

$$= 2\ln\left|\frac{2x+3}{3x+5}\right| + c$$

2. Evaluate the indefinite integral

$$\int \left(x - \frac{8x}{x^2+8}\right)dx \quad \int \frac{x^3}{x^2+8}dx$$

$$= \frac{x^2}{2} - 4\ln(x^2+8) + c$$

3.a) Find the partial fraction decomposition of $\dfrac{4x^2+73x+168}{(x+4)^2(x-8)}$

$$\frac{A}{(x+4)^2} + \frac{B}{x+4} + \frac{C}{x-8}$$

$$(B+C)x^2 + (A-4B+8C)x - 8A - 32B + 16C = 4x^2 + 73x + 168$$

$$\frac{5}{(x+4)^2} - \frac{3}{x+4} + \frac{7}{x-8}$$

b) Use the result of part (a) to evaluate the integral $\displaystyle\int \frac{4x^2+73x+168}{(x+4)^2(x-8)}dx.$

$$5\int \frac{dx}{(x+4)^2} - 3\int \frac{dx}{x+4} + 7\int \frac{dx}{x-8}$$

$$-5(x+4)^{-1} - 3\ln|x+4| + 7\ln|x-8| + C$$

# 6525 Harder Partial Fractions

1. Find an approximate value of the integral $\int_1^4 (4x - x^2)dx$ using the midpoint rule with $n = 6$.

$1 \quad 5/4 \quad 3/2 \quad 7/4 \quad 2 \quad 9/4 \quad 5/2 \quad 11/4 \quad 3 \quad 13/4 \quad 7/2 \quad 15/4 \quad 4$

$f(\frac{5}{4}) = \frac{55}{16}$, $f(\frac{7}{4}) = \frac{63}{16}$, $f(\frac{9}{4}) = \frac{63}{16}$, $f(\frac{11}{4}) = \frac{55}{16}$, $f(\frac{13}{4}) = \frac{39}{16}$

$f(\frac{15}{4}) = \frac{15}{16}$ $\quad \left[\frac{55}{16} + \frac{63}{16} + \frac{63}{16} + \frac{55}{16} + \frac{39}{16} + \frac{15}{16}\right] = \frac{290}{16}$

$\frac{1}{2}\left[\frac{290}{16}\right] = \frac{145}{16} = 9.0625$

ERROR $= 0.0625$

2. Convert the following definite integral into another definite integral using the substitution $u = 5x + 9$.

$$\int_0^8 x(5x+9)^{3/2}dx.$$

when $x=0$ $\quad u = 9$

when $x=8$ $\quad u=49$

$$\frac{1}{25}\int_9^{49}(u^{5/2} - 9u^{3/2})du$$

3. Evaluate the indefinite integral $\int \frac{120x}{(16x^2+9)(2x+3)}dx.$

$$\int \left[\frac{32x+12}{16x^2+9} + \frac{-4}{2x+3}\right]dx$$

$$= \int \frac{du}{u} + \int \frac{du}{1+u^2} - 2\int \frac{du}{u}$$

$$= \ln(16x^2+9) + \arctan\left(\frac{4x}{3}\right) - 2\ln|2x+3| + C$$