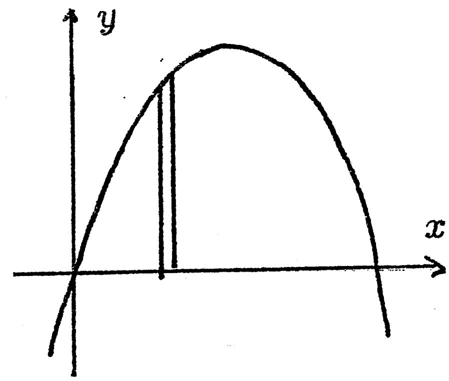
6541 First Moments

1. The region bounded by the curve $y = 5x - x^2$ and the x axis is covered by a lamina of constant density ρ. Find M_x, the first moment of this lamina about the x axis. Find M_y, the first moment of this lamina about the y axis. Find the mass of the lamina. Find the center of mass. Find the moment about the line $y = -3/2$.



$$\Delta M_x = \rho\left(\tfrac{1}{2}y^2\right)\Delta x$$

$$M_x = \frac{\rho}{2}\int_0^5 (5x - x^2)^2\,dx = \frac{625}{12}\rho$$

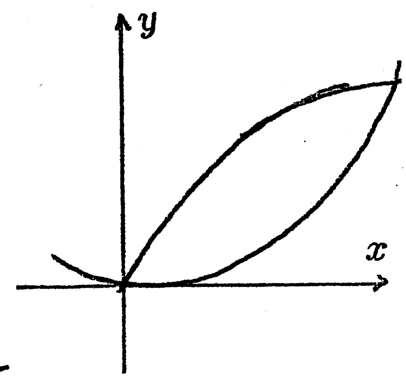
$$M_y = \rho\int_0^5 x(5x - x^2)\,dx = \frac{625}{12}\rho$$

$$\text{MASS} = \rho\int_0^5 (5x - x^2)\,dx = \frac{125}{6}\rho$$

$$(\bar{x}, \bar{y}) = \left(\frac{5}{2}, \frac{5}{2}\right)$$

Moment About $y = -3/2$ is $\frac{625}{12}\rho + \frac{3}{2}\,\frac{125}{6}\rho = \frac{1000}{12}\rho$

2. The region bounded by the curves $y = \sqrt{2}x^2$ and $y = 4\sqrt{x}$ is covered by a lamina of constant density ρ. Find M_x and M_y for this lamina. Find the mass of the lamina. Find the center of mass. Find the first moment about the line $x = -5$.



$$M_x = \frac{\rho}{2}\int_0^2 \left[(4\sqrt{x})^2 - (\sqrt{2}x^2)^2\right]dx = \frac{48}{5}\rho$$

$$M_y = \rho\int_0^2 x\left[4\sqrt{x} - \sqrt{2}x^2\right]dx = \frac{12\sqrt{2}\rho}{5}$$

$$\text{MASS} = \rho\int_0^2 (4\sqrt{x} - \sqrt{2}x^2)\,dx = \frac{8\sqrt{2}}{3}\rho$$

$$(\bar{x}, \bar{y}) = \left(\frac{9}{10}, \frac{9\sqrt{2}}{5}\right)$$

Moment About $x = -5$ is

$$\left(5 + \frac{9}{10}\right)\frac{8\sqrt{2}}{3}\rho = \frac{236\sqrt{2}}{15}\rho$$

6542 Variables Separable

1. Solve the initial value problem $\frac{dy}{dx} = 2y + 6$ and $y(0) = 3$. Solve for y.

Note $c = 0$ $y = -3$ is A Solution.

$$\frac{dy}{2y+6} = dx \qquad y \neq -3$$

$$\ln|2y+6| = 2x + \ln c \qquad c \neq 0$$

$$2y + 6 = ce^{2x}$$

when $x = 0$ $y = 3$

$c = 12$

$$y = -3 + 6e^{2x}$$

2. Solve the initial value problem $\frac{dy}{dx} = \frac{8x(3y+1)}{3(4x^2+5)}$ and $y(1) = 2$.

$$\frac{3dy}{3y+1} = \frac{8x\,dx}{4x^2+5}$$

$$\ln|3y+1| = \ln(4x^2+5) + \ln c$$

$$3y + 1 = c(4x^2+5)$$

when $x = 1$ $y = 2$

$7 = 9c$

$$y = -\frac{1}{3} + \frac{7}{27}(4x^2+5)$$

3. The family of curves $y = cx^2$ is the general solution to the differential equation $\frac{dy}{dx} = \frac{2y}{x}$. Sketch the four members of this family corresponding to $c = -1, 1/2, 1, 2$.