

6551 Euler Method

Use the Euler method with the indicated step size to compute approximate values of y for the solution of the following initial value problem. Compute y_1, y_2, y_3 , and y_4 .

1. $y' = 2x + y$ $y(1) = 2$ $h = 0.4$

$$X_0 = 1 \quad y_{n+1} = y_n + \frac{h}{10} (2X_n + y_n)$$

$$X_1 = 1.4$$

$$X_2 = 1.8$$

$$X_3 = 2.2$$

$$y_0 = 2$$

$$y_1 = 3.6$$

$$y_2 = 6.16$$

$$y_3 = 10.064$$

$$y_4 = 15.8496$$

2. $y' = y - 2x$ $y(1) = 5$ $h = 0.4$

$$X_0 = 1 \quad y_{n+1} = y_n + \frac{h}{10} (y_n - 2X_n)$$

$$X_1 = 1.4$$

$$X_2 = 1.8$$

$$X_3 = 2.2$$

$$y_0 = 5$$

$$y_1 = 6.2$$

$$y_2 = 7.56$$

$$y_3 = 9.144$$

$$y_4 = 11.0416$$

3. On side of a tank that is filled with water is shown below. Find the hydrostatic force on this side of the tank. The lengths are given in meters. Recall that the density of water is 9800 Newtons/meter³.

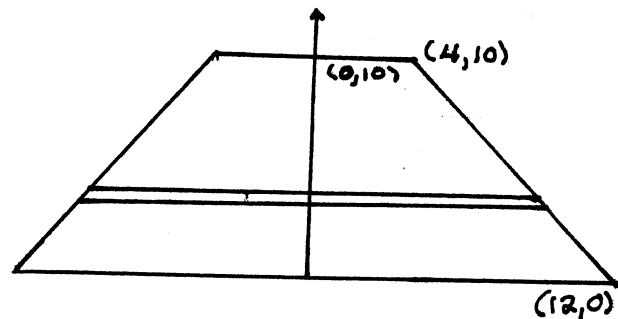
$$y - 10 = -\frac{5}{4} (x - 4)$$

$$x = -\frac{4}{5} y + 12$$

$$\Delta F = \rho (10 - y) (2x) \Delta y$$

$$\text{Force} = \int_0^{10} (10 - y) \left(-\frac{8}{5} y + 24\right) dy$$

$$= \frac{2800}{3} \rho = 9,146,667 \text{ Newtons}$$



6552 Logistic Equations

1. First find the general solution of the differential equation. Second, solve the two initial value problems. Solve for y .

(a) $\frac{dy}{dx} = (2+y)(8-y)$ and $y(0) = 6$

$y(0) = 6$ gives $C = 4$

(b) $\frac{dy}{dx} = (2+y)(8-y)$ and $y(0) = 10$

$$\ln|2+y| - \ln|8-y| = 10x + \ln C$$

$$y = \frac{32e^{10x} - 2}{1 + 4e^{10x}}$$

$$\left| \frac{2+y}{8-y} \right| = Ce^{10x}$$

$$= \frac{32 - 2e^{-10x}}{4 + e^{-10x}}$$

$$\frac{2+y}{8-y} = Ce^{10x}$$

$y \neq 8$
 $y \neq -2$

$y(0) = 10$ gives $C = -6$

$$y = \frac{8Ce^{10x} - 2}{1 + Ce^{10x}}$$

$$y = \frac{2 + 48e^{10x}}{6e^{10x} - 1}$$

2. Find the indefinite integral $\int \frac{x^2}{\sqrt{4x+9}} dx$.

Let $u^2 = 4x+9$ $x = \frac{u^2-9}{4}$ $dx = \frac{u}{2} du$

$$\frac{1}{32} \int (u^4 - 18u^2 + 81) du$$

$$= \frac{1}{32} \left[\frac{u^5}{5} - 6u^3 + 81u \right] + C$$

$$= \frac{(4x+9)^{5/2}}{160} - \frac{3}{16} (4x+9)^{3/2} + \frac{81}{32} (4x+9)^{1/2} + C$$

the substitution $u = 4x+9$ gives

$$\frac{1}{64} \int (u^{3/2} - 18u^{1/2} + 81u^{-1/2}) du$$