6531 Arc Length

1. Find the arc length of the curve whose equation is $y = (1/8)x^2 - \ln x$ from the point $(1, 1/8)$ to the point $(4, 2 - \ln 4)$.

$$1 + \left(\frac{dy}{dx}\right)^2 = 1 + \frac{1}{16}x^2 - \frac{1}{2} + \frac{1}{x^2} = \left(\frac{x}{4} + \frac{1}{x}\right)^2$$

$$\int_1^4 \left(\frac{x}{4} + \frac{1}{x}\right)dx = \frac{x^2}{8} + \ln x \Big|_1^4 = \frac{15}{8} + \ln 4$$

2. Find the arc length of the curve whose equation is $y = (1/9)(x^2 + 6)^{3/2}$ from the point where $x = 0$ to point where $x = 6$.

$$\frac{dy}{dx} = \frac{1}{9}\,\frac{3}{2}\,(x^2+6)^{1/2}(2x)$$

$$= \frac{x}{3}(x^2+6)^{1/2}$$

$$1 + \left(\frac{dy}{dx}\right)^2 = \frac{1}{9}(x^2+3)^2$$

$$\int_0^6 \frac{1}{3}(x^2+3)\,dx$$

$$= 30$$

3. Transform the following definite integral into a definite integral of equal value with variable of integration θ using the substitution $3x = 2\sin\theta$

$$\int_{\sqrt{2}/3}^{\sqrt{3}/3} \frac{x^2\,dx}{(4 - 9x^2)^{3/2}}.$$

when $x = \frac{\sqrt{2}}{3}$ $\sin\theta = \frac{\sqrt{2}}{2}$

$\theta = \pi/4$

when $x = \frac{\sqrt{3}}{3}$ $\sin\theta = \frac{\sqrt{3}}{2}$

$\theta = \pi/3$

$$\frac{1}{27}\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \tan^2\theta\, d\theta$$

4. Transform the following definite integral into a definite integral of equal value with variable of integration u using the substitution $u = 3x + 10$.

$$\int_2^5 \frac{x}{(3x + 10)^{1/2}}dx.$$

when $x = 2$, $u = 16$

when $x = 5$, $u = 25$

$$\frac{1}{9}\int_{16}^{25} \left(u^{1/2} - 10u^{-1/2}\right)du$$

6532 Easy Error Bounds

1. Suppose that $f(x)$ is such that $|f''(x)| \leq 8$ for $1 \leq x \leq 6$. Find the error bound for the integral $\int_1^6 f(x)dx$ when $n = 50$. Recall that the actual error E_T when using the trapezoid rule is bounded by $|E_T| \leq \frac{K(b-a)^3}{12n^2}$.

$$\frac{K(b-a)^3}{12n^2} = \frac{8(6-1)^3}{12(50)^2} = \frac{1}{30} \qquad |E_T| \leq \frac{1}{30}$$

2. Suppose that $f(x)$ is such that $|f''(x)| \leq 15$ For $x \geq 0$. Find the error bound for the integral $\int_2^{10} f(x)dx$ when $n = 200$. We denote the actual error by E_T.

$$\frac{K(b-a)^3}{12n^2} = \frac{15(10-2)^3}{12(200)^2} = \frac{2}{125} \qquad |E_T| \leq 0.016$$

3. Let $f(x) = 18 + 10x + 3x^2 - 2x^3$. Show that $|f''(x)| \leq 18$ for $0 \leq x \leq 2$.

$f''(x) = 6 - 12x$ $\qquad |f''(x)| \leq 18$

$f''(x)$ is linear

$f''(0) = 6$

$f''(2) = -18$

4. Find the error bound for E_T, the error when using the trapezoid rule, for the integral $\int_0^2 (18 + 10x + 3x^2 - 2x^3)dx$ when $n = 150$.

$$\frac{K(b-a)^3}{12n^2} = \frac{18(2)^3}{12(150)^2} = \frac{1}{1875}$$

$$|E_T| \leq \frac{1}{875} < 0.00054$$