

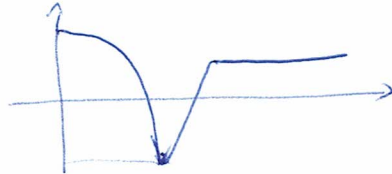
9911.4 Practice with "choppers"

1. Let

$$f(t) = \begin{cases} 4 - t^2 & \text{for } 0 \leq t < 3, \\ 2t - 11 & \text{for } 3 \leq t < 6, \\ 1 & \text{for } t \geq 6. \end{cases}$$

Express $f(t)$ in terms of the unit step function $H(t)$. Also sketch a graph of $f(t)$.

$$f(t) = (4 - t^2)H(t) + (t^2 + 2t - 15)H(t-3) - 2(t-6)H(t-6)$$



2. Let

$$f(t) = \begin{cases} 0 & \text{for } 0 \leq t < 2 \\ t^2 - 4 & \text{for } 2 \leq t < 4 \\ -2t + 16 & \text{for } 4 \leq t < 6 \\ 0 & \text{for } t \geq 6. \end{cases}$$

Express $f(t)$ in terms of the unit step function $H(t)$. Also sketch a graph of $f(t)$.

$$f(t) = (t^2 - 4)H(t-2) + (-t^2 - 2t + 20)H(t-4) + (2t - 16)H(t-6)$$

3. Solve the IVP $y'' + 4y' + 13y = 78e^{-4t}$, $y(0) = -3$, $y'(0) = 8$ using Laplace transforms.

$$\begin{aligned} Y(s) &= \frac{3s^2 + 16s - 62}{(s+4)(s^2 + 4s + 13)} = \frac{6}{s+4} - \frac{9s+4}{s^2 + 4s + 13} \\ &= \frac{6}{s+4} - \frac{9s+4}{(s+2)^2 + 3^2} \end{aligned}$$

$$y(t) = 6e^{-4t} - 9e^{-2t} \cos(3t) + \frac{14}{3}e^{-2t} \sin(3t).$$