Worksheets

05/01/2018

1. Solve the following initial-value problem using **Laplace transform**

$$\vec{\mathbf{x}}' = \begin{bmatrix} 3 & -3 \\ 2 & -2 \end{bmatrix} \vec{\mathbf{x}} + \begin{bmatrix} 4 \\ -1 \end{bmatrix}, \qquad \vec{\mathbf{x}}(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

2. Solve the following initial-value problem using **undetermined coefficient method** (i.e. superposition method)

$$\vec{\mathbf{x}}' = \begin{bmatrix} 0 & -1 \\ 3 & 4 \end{bmatrix} \vec{\mathbf{x}} + \begin{bmatrix} t \\ -4t-2 \end{bmatrix}, \qquad \vec{\mathbf{x}}(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

3. Determine a closed-form solution for the Fibonacci iterative equation

$$y_0 = 1, \; y_1 = 1, \; y_{n+1} = y_n + y_{n-1}$$

4. A certain horse runs in a three-horse race. If he wins a race, he has probability 1/2 of winning, 1/3 coming in second, and 1/6 coming in third the next race. If he comes in second on a race, he has probability 1/3 of winning, 1/6 coming in second, and 1/2 coming in third the next race. If he comes in third on a race, he has probability 1/3 of winning, 1/3 coming in second, and 1/3 coming in third the next race. Let x_n, y_n, z_n respectively be the probability the horse coming in first, second, third in the n'th race. Then

$$\begin{bmatrix} x_{n+1} \\ y_{n+1} \\ z_{n+1} \end{bmatrix} = A \begin{bmatrix} x_n \\ y_n \\ z_n \end{bmatrix}$$

(a) Find the 3×3 matrix A.

(b) The horse comes in second on the first race. Find the probability that he wins / comes in second / third on race n. [Given that the eigenvalues of matrix A are 1, 1/6, -1/6.]

(c) What is the limit of x_n as $n \to \infty$?

5. Solve the initial-value problem

$$\vec{\mathbf{x}}' = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 2 \\ 0 & -2 & 0 \end{bmatrix} \vec{\mathbf{x}}, \qquad \vec{\mathbf{x}}(0) = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$