Worksheets

05/03/2018

1. Suppose that
$$y(t) = 3e^{-2t}\cos t - 2e^{-2t}\sin t + \sqrt{2}\cos(5t - \delta)$$
is the solution of a mass-spring system
$$mx'' + bx' + kx = F_0 \cos \omega_f t.$$

   (a) What part of the solution is the transient solution? What part is the steady-state solution?

   (b) If the mass is 1 kg, what are the damping constant b and the restoring constant k?

   (c) Is the system underdamped, critically damped, or overdamped?

   (d) What is the time-varying amplitude of the transient solution?

   (e) Find the angular frequency ω_f of the forcing function. Find the forcing amplitude F_0.

2. Determine whether the following matrix is diagonalizable. If it is, diagonalize it (i.e. determine an invertible matrix P and a diagonal matrix D such that $A = PDP^{-1}$).

$$A = \begin{bmatrix} 4 & 2 & 0 \\ -1 & 1 & 0 \\ 0 & 1 & 2 \end{bmatrix}$$

3. (a) Check if the following set of vectors are linearly independent. If not, express one vector as a linear combination of the others.

$$v_1 = (1,0,1,1),\ v_2 = (-1,1,0,0),\ v_3 = (2,1,-1,1),\ v_4 = (-1,0,3,1)$$

(b) Recall that the set $C(\mathbb{R})$ of continuous functions on the real line $\mathbb{R}$ is a vector space (in which each vector is a continuous function). Check if the following set of functions are linearly independent.

$$f_1(t) = t, \quad f_2(t) = \cos t$$

4. Suppose two equal masses $m_1 = m_2 = 1$ are attached to three springs, where two outside springs are attached to walls. The spring constants from the left to right are respectively $k_1 = 1$, $k_2 = 2$, $k_3 = 1$. Denote by $x(t)$ and $y(t)$ the positions of the respective masses m_1 and m_2 from their respective equilibrium positions.

   The system is set in motion by holding the left mass in its equilibrium position while at the same time pulling the right mass to the right of its equilibrium a distance $y(0) = 2$.

   (a) Write the differential equations for $x(t)$ and $y(t)$ together with the initial conditions.

   (b) Determine the Laplace transforms of $x(t)$ and $y(t)$.