

## HOMEWORK #2 (DUE FRIDAY, OCT. 7).

9/26/2011

**Note:** This homework is pretty long, so it is a good idea to start early. As before, turn in only the starred problems.

Problems 19, 20, 24\*, 25\*, 26\*, 28\*, 29, 30\*, 34, 38 (a), (b), (c) in Chapter I in Lang.

### Additional problems:

1\*) Let  $G$  be a subgroup of  $S_n$ .

(i) If  $G \cap A_n = \{id\}$ , then  $|G| \leq 2$ .

(ii) If  $|G| > 2$  and  $G$  is simple, then  $G \subset A_n$ .

(iii) If  $n \geq 5$ , then  $S_n$  has no subgroup of index  $m$  with  $2 < m < n$ .

(iv) If  $n \geq 5$ , then  $A_n$  has no subgroup of index  $m$  with  $2 \leq m < n$ .

2\*) Prove that there are no simple groups of order 90.

3\*) Show that every group of order 231 is the direct product of a group of order 11 and a group of order 21.

4\*) Give an example of a finite group  $G$  having  $p$ -Sylow subgroups  $P, Q$ , and  $R$  (for some prime  $p$ ) with  $P \cap Q = \{e\}$  and  $P \cap R \neq \{e\}$ .