MATH 8202 HOMEWORK #3 (DUE MONDAY, APRIL 29).

4/11/2013

Turn in the starred problems only.

Problems from Lang's book:
Problems 3^* and 7^* in Chapter VII.
Problems 1^* and 3 in Chapter VIII.

Additional problems

1^*) (i) Let $A \subset B \subset C$ be commutative rings. Assume that A is Noetherian, C is finitely generated as an A-algebra and that C is integral over B. Show that B is a finitely generated A-algebra.

(ii) Let A be a Noetherian ring, B a finitely generated A-algebra, and G a finite group of automorphisms of B over A. Prove that the subring of G-invariants $B^G = \{b \in B \mid \sigma(b) = b \; \forall \sigma \in G\}$ is a finitely generated A-algebra.

2^*) Let A be a commutative ring and let $f_1, \dots f_n \in A[X]$ be monic polynomials of positive degree.

(i) Show that there exists a ring extension $A \subset A'$, with A' finitely generated as an A-module, such that each f_i has $\deg(f_i)$ roots in B.

(ii) Show that there exists a ring extension $A \subset A''$ such that A'' is integral over A and every monic polynomial (of positive degree) has a root in A''.

(iii) Let $A \subset B$ be a ring extension and let C be the integral closure of A in B. If $f, g \in B[X]$ are monic polynomials such that fg has coefficients in C, then f and g have coefficients in C.

3^*) Let A be a commutative ring. Prove that A is Noetherian iff every prime ideal is finitely generated.