

# **PROBLEM SETS**

## **Math 8386: Calculus of Variations**

### **FIRST PROBLEM SET**

Due M 17 February 2014.

1. Prove Lemma 2.2.1, p.133 of Jost & Li-Jost (5 points).
2. Prove that  $\|f\|_*$ , defined in equation (2.2.1), p. 133, is a norm on the dual space  $V^*$  (5 points).
3. Prove W.H.Young's inequality: if  $a, b, p, q \geq 0$ ,  $\frac{1}{p} + \frac{1}{q} = 1$ , then

$$ab \leq \frac{a^p}{p} + \frac{b^q}{q}.$$

(5 points).

4. Exercise #2.1 on p. 157 (10 points).
5. Exercise #2.2 on p. 157 (10 points).
6. Exercise #2.3 on p. 157 (10 points).