8701 – Fall 2012 – Problem Set 7

1. Let Ω be an open, connected set and let $\gamma_a(t) \equiv a$ denote the constant curve that is identically equal to $a \in \Omega$ for $t \in [0,1]$. Show that if a (smooth) closed curve γ is homotopic to γ_a, then γ is homotopic to the constant curve $\gamma_b \equiv b$ for any other point $b \in \Omega$. (Thus, when we say that a closed curve is "homotopically trivial" we need not specify a point in Ω to which it deforms.)

2. Show that if we change the definition of *homotopic* given in class, by removing the restriction that $\Gamma(0,t) = \Gamma(1,t)$ for all $t \in [0,1]$, then we can find two curves which are "homotopic" (in this altered sense) in $\mathbb{C} - \{0\}$, but have different line integrals for some function f on $\mathbb{C} - \{0\}$. (Thus, the general form of Cauchy's theorem would be false, as stated, with this modified definition of homotopy.)