

8701 – Fall 2012 – Problem Set 8

1. Find the residues at all singularities for the following functions:

a) $\frac{z}{z^4 + 1}$

b) $\frac{\sin z}{z^2(\pi - z)}$

c) $\frac{ze^{iz}}{(z - \pi)^2}$

d) $\frac{z^3 + 5}{(z^4 - 1)(z + 1)}$

2. Let C denote the unit circle, traversed counter-clockwise. Compute each of the integrals:

$$\int_C \frac{e^{\pi z}}{4z^2 + 1} dz, \quad \int_C \frac{e^z}{(z^2 + z - 3/4)^2} dz$$

3. Riemann integrals of a real variable θ of the form

$$\int_0^{2\pi} f(\cos \theta, \sin \theta) d\theta$$

can sometimes be solved by changing variables to a complex contour integration.

Indeed, setting $z = e^{i\theta}$ then as θ runs from 0 to 2π , z traverses the unit circle counterclockwise. Moreover,

$$\cos \theta = \frac{z^2 + 1}{2z}, \quad \sin \theta = \frac{z^2 - 1}{2iz}, \quad d\theta = \frac{dz}{iz}.$$

Show that such a change of variables allows one to evaluate:

$$\int_0^{2\pi} \frac{d\theta}{2 + \sin \theta}$$

via a complex contour integral, upon using the residue theorem.

4. Problems from Ahlfors, Section 4.5.2: **p. 154, problems 1 and 2.**