MATH 8702
Additional Problems for Homework Assignment #2
Due 02/15

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1. Let $f(z)$ be a holomorphic function in $U = \{z : Re(z) > 0\}$ with $|f(z)| < 1$ for all $z \in U$. If $f(1) = 0$, find maximum possible value of $|f(2)|$.
2. Let $\mathcal{F} = \{f : f \text{ is holomorphic in } \mathbb{D}, f(0) = 0, diameter(f(\mathbb{D})) \leq 2\}$. Prove that $\mathcal{F}$ is both normal family and compact.
3. Let f be a holomorphic function on the unit disc $\mathbb{D}$ with $f(0) = 0$ and $|f(z)| < 1$ for all $z \in \mathbb{D}$. Define the sequence of functions f_n on the unit disc by $f_n(z) = \underbrace{f(f(\cdots f(z))\cdots)}_{n}$. Prove that if $f_n(z)$ converges to $h(z)$ for all $z \in \mathbb{D}$, then either $h(z) = z$ or $h(z) = 0$.