Math 8501, Fall 2014

Theory of Ordinary Differential Equations

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— *Optional exercises — can be turned in on Friday, 9/5* —

(i) An exercise in abstraction:

(a) Show that $Y = C^0([0,1])$ is a Banach space when equipped with the norm

$$\|u\|_{C^0} = \sup_{t\in[0,1]} |u(t)|.$$

(b) Show that $X = C^1([0,1]) \cap \{u|u(0) = 0\}$ is a Banach space when equipped with the norm

$$\|u\|_{C^1} = \sup_{t\in[0,1]} (|u'(t)| + |u(t)|).$$

(c) Show that the operator

$$L : X \to Y, \quad u \mapsto u',$$

is linear and bounded.

(d) Show that L is bounded invertible and describe the inverse.

(ii) Find the solution to $x' = Ax \in \mathbb{R}^2$, $A = \begin{pmatrix} 1 & \mu \\ 0 & 1 \end{pmatrix}$, with $x(0) = (0,1)^T$, for any $\mu \in \mathbb{R}$.