Math 8501, Fall 2014

Theory of Ordinary Differential Equations

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— *Homework 2* —

(1) Consider the initial-value problem $\dot{u} = u(1-u), u(0) = u_0$.

- (i) Find the explicit solution $u(t;u_0)$ for $u_0 \in (0,1)$ and for $u_0 = 1$. Differentiate the explicit solution with respect to t and with respect to u_0.
- (ii) Find an initial-value problem for $\partial_{u_0} u(t;u_0)$ and for $\partial_t u(t;u_0)$ and find the explicit solutions of the initial-value problem. Compare your solution with the direct calculation from (i).

(2) Consider $\dot{x} = f(x), x(0) = x_0$ and assume that $f(x) \cdot x \leq 0$ for all x with $|x|$ sufficiently large. Show that solutions exist for all $t \geq 0$.

(3) Which of the following initial value problems possess solutions for all $t \in \mathbb{R}$ — eplain why!

- (i) $\dot{x} = \sin(x)$, $x(0) = x_0$;
- (ii) $\dot{x} = x - x^3$, $x(0) = 2$;
- (iii) $\dot{x} = x - x^3$, $x(0) = 1/2$;
- (iv) $\dot{x} = \frac{4x^3+y}{1+x^2+y^2}$, $\dot{y} = \frac{y^5-x^5}{x^4+y^4+1}$;

Homework is due on Friday, October 10, in class