Math 8501, Fall 2014

Theory of Ordinary Differential Equations

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— Homework 3, due on Friday, October 24, in class —

(1) Let M be a compact set and define

$$\omega(M) = \bigcap_{T>0} \overline{\bigcup_{t\geq T} \Phi_t(M)}.$$

Show that $\omega(M) \supseteq \bigcup_{x\in M} \omega(x)$ and find an example where the inclusion is strict.

(2) Consider $u' = u^2 - vu^3, v' = 0$.

- (i) Draw a phase portrait.
- (ii) Find the points for which the maximal positive time of existence is finite, $t^+(u, v) < \infty$.
- (iii) Show that t^+ is not a continuous function when considered in $\mathbb{R}_+ \cap \infty$ with the usual topology of convergence to infinity.

(3) Consider the equation $z' = z^2$ on the Riemann sphere, $z \in \mathbb{C}$.

- (i) Perform the change of coordinates $\zeta = 1/z$ and find the vector field at $z = \infty$.
- (ii) Find (explicitly) all solutions and plot a phase portrait.
- (iii) Show that all solutions converge to the origin.
- (iv) Show that the origin is not Lyapunov stable.

(4) Sketch the phase portrait (in particular all heteroclinic and homoclinic orbits) to $x'' = -V'(x)$, with $V(x) = x^6 - 0.03x^5 - 4x^4 + 4x^2 + 0.2x$,