Math 8501, Fall 2014

Theory of Ordinary Differential Equations

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— *Homework 4* —

(1) Consider the Lotka-Volterra model for competing species

$$u' = u(1 - u - bv)$$
$$v' = v(1 - v - au)$$

for parameter values $0 < a \leq b$ ($a \geq b$ can be obtained by interchanging u and v).

(i) Find all equilibria with $u, v \geq 0$ depending on a, b; you should find changes when crossing the curves $a = 1$, $b = 1$, or $ab = 1$ in the (a, b)–plane.

(ii) Sketch the nonlinear phase portrait for the nonlinear system in neighborhoods of equilibria on the coordinate axes using the information on the linearization at the equilibria.

(iii) For the equilibrium $u = 1, v = 0$, compute the spectral projections on the unstable and on the stable eigenspace (when the equilibrium is hyperbolic).

(iv) *Optional:* Also draw phase portraits near the equilibria with $u, v > 0$. There's a phase portrait for each $a, b > 0$, which changes at the curves listed in (i).

(2) Show that $e^{A+B} = e^A e^B$ when $AB = BA$. Give an example that this is not true when $AB \neq BA$. *Optional:* Does $e^{A+B} = e^A e^B$ imply $AB = BA$?

(3) For a hyperbolic matrix A show that the unstable subspace E^u is asymptotically stable as an invariant set.

Homework is due on Friday, November 7, in class