Math 8502, Fall 2015

Dynamical Systems and Differential Equations II

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— Homework 1 —

(1) Consider perturbations of a length-n Jordan block, $A(\mu) = N_n + B(\mu)$, where $N_{j,k} = 1$, $j = k-1$, $N_{j,k} = 0$, otherwise, and $B_{j,j-\ell} = b_\ell(\mu)$ for all j and all $\ell \geq 0$, $B_{j,k} = 0$ otherwise (this is sometimes called the Arnold versal unfolding). Suppose that all b_j are analytic functions, $b_j(0) = 0$, and $b'_{n-1}(0) \neq 0$. Find all eigenvalues to leading order in μ near $\mu = 0$.

(2) Consider the Hamiltonian system

$$\dot{x} = \partial_y H(x,y), \quad \dot{y} = -\partial_x H(x,y)$$

with analytic H, $H(x,y) = x^4 + 5xy + y^3 + \mathrm{O}(x^6 + y^6)$. Find all trajectories that converge to the origin as $t \to +\infty$ or $t \to -\infty$, in the form $x = \varphi(y)$ and give first-order expansions for φ.

(3) Consider a ring of three cells with anti-diffusive coupling,

$$\dot{u}_j = f(u_j) + d(u_{j+1} - 2u_j + u_{j-1}), \quad j \in \mathbb{Z}/3\mathbb{Z}, \quad d < 0.$$

Suppose $f(0) = 0, f'(0) = -1, f''(0) \neq 0$. Find the parameter value $d_* \in (f'(0), 0)$ where the trivial equilibrium $u_j = 0$, $j = 0, 1$, is not hyperbolic. Now restrict to $u_1 = u_2$ and find all solutions bifurcating from the trivial solution for $d \sim d_*$.

(4) Show that two-parameter families of matrices $A(\mu_1, \mu_2)$, $\mu_1, \mu_2 \in [0,1]$ can be perturbed by an arbitrarily small amount so that the kernel of the matrix is at most one-dimensional for all μ_j.

Due February 6.