Math 8590 Spring 2014

Homework assignment 2

due April 25, 2014

Let $u(x,t)$ be a mild solution of the Navier-Stokes equation in $\mathbf{R}^3 \times (0,T)$, with initial condition $u(\cdot,0) = u_0 \in L^2_x \cap L^\infty_x$. (In class we showed that under these assumptions u is smooth away from the initial time $t = 0$ and the energy identity is satisfied. If T is the maximal time of existence of the mild solution, the estimates on $\sup_x |u(x,t)|$ and other higher-regularity quantities will deteriorate as t approaches T. We do not wish to rule out this possibility from our considerations.) In class we sketched a proof of the following statement:

For each $\varepsilon > 0$ there exists $R > 0$ such that for our solution u we have:

$$\int_{|x|>R} |u(x,t)|^2 \, dx \le \varepsilon \tag{1}$$

for all $t \in (0,T)$.

Give a detailed proof of the statement.