Theory of Probability and Measure Theory – Math 8651

Homework #2

Problems 7.2, 7.8. In Problem 7.14, show not what is asked there but that any Borel set is regular. Note that $\mathcal{B}$ there is our $\mathcal{B}(\mathbb{R})$. Additional problems:

A) Prove that if $\mathcal{U}$ is both a λ-system and a π-system then it is a σ-field.

B) Prove that if μ and ν are σ-finite measures on a measurable space $(\Omega), \mathcal{F})$, $\mathcal{F} = \sigma(\mathcal{E})$, $\mathcal{E}$ is a π-system, and $\mu = \nu$ on $\mathcal{E}$, then $\mu = \nu$ on $\mathcal{F}$.

In the following two problems, $\mathcal{B}$ is a collection of subsets of Ω such that $\emptyset \in \mathcal{B}$ and f is a nonnegative function on $\mathcal{B}$ such that $f(\emptyset) = 0$.

C) We call $A \subset \Omega$ an f-null set if $f^*(A) = 0$. Prove that f-null sets are f^*-sets.

D) Prove that if $\mathcal{B}$ is a σ-field and f is σ-additive on $\mathcal{B}$ then for any $A \subset \Omega$ there exists $B \in \mathcal{B}$ such that $A \subset B$ and $f^*(A) = f^*(B)$.