Theory of Probability and Measure Theory – Math 8651

Homework #7

Problems 4.20, 9.7, 14.1, 14.15, 14.42. Additional problems:

A) (Similar to Problem 12.17) Find the limit of $\left(\prod_{k=1}^{n} X_k\right)^{1/n}$, where X_k's are i.i.d. uniformly distributed on $[0,1]$.

B) Let X be a Polish space and $K \subset X$. Prove that the following are equivalent.

(i) K is a closed, totally bounded set (i.e. for every $\varepsilon > 0$ there exists a finite set $A = \{x_1, ..., x_n\}$, called an ε-net, such that every point of K is in the ε-neighborhood of at least one point in A).

(ii) For every sequence of points $x_n \in K$, there is a subsequence that converges to an element of K.