Theory of Probability and Measure Theory – Math 8652

Homework #2

1) (Problem 13.23 in the textbook) Let ϕ be the moment generating function of a $[0, \infty]$-valued randome variable X. Show that ϕ is continuous on $(0, \infty)$ and if $P(X < \infty) = 1$, then on $[0, \infty)$.

2) Let $X_1, X_2, \ldots$ be nonnegative independent random variable having the same distribution. Assume that $P(X_1 > 0) > 0$. Prove that

$$\sum_{n=1}^{\infty} X_n = \infty \quad \text{a.s.}$$

3) (Problem 13.40 in the textbook) Let X_n, $n = 1, 2, \ldots$, be iid nonnegative random variables and let N be a $\{\infty, 1, 2, \ldots\}$-valued random variable independent of $(X_1, X_2, \ldots)$. Express the moment generating function of $S = X_1 + \ldots + X_N$ through the moment generating function of X_1 and the probability generating function of N. Then find ES and $\text{Var}S$.

4) (Theorem 14.19 in the textbook) A sequence of distributions on $[0, \infty)$ converges to a distribution Q on $[0, \infty)$ if and only if the sequence of the corresponding moment generating functions converges pointwise on $[0, \infty)$ to a function ϕ that is continuous at zero. Moreover, ϕ is the moment generating function of Q.