Theory of Probability and Measure Theory – Math 8652

Homework #3

1) Show that the function $\sqrt{(1-t^2)_+}$ is not positive definite.

2) Show that the product of two completely monotone functions is completely monotone.

3) Compute

$$\lim_{n\to\infty} \sum_{k=0}^{n} \frac{(k+1)n^k}{(1+n)^{k+2}}.$$

4) (Problem 12.26 in the textbook) Let X_n, $n \geq 1$, be iid real-valued and set $S_n = X_1 + ... + X_n$,

$$c = \overline{\lim_{n\to\infty}} \frac{S_n}{n}.$$

Show that $P(\exists$ infinitely many n such that $S_n > cn)$ equals 0 or 1.