**Theory of Probability and Measure Theory – Math 8652**

Homework #6

1) Let $\xi_n$ be summable $\mathcal{F}_n$-measurable random variables given for $n = 0, 1, ...$ Assume that for any bounded stopping times $\tau \geq \sigma$ we have $E\xi_\tau \geq E\xi_\sigma$. Prove that $(\xi_n, \mathcal{F}_n)$ is a submartingale.

2) (Problem 24.24 in the textbook) Consider a random walk $S_0 = 0$, $S_n = X_1 + ... + X_n$, where $X_k$'s are iid on $\mathbb{R}$ whose step size has mean zero. Set

$$\tau = \inf\{n \geq 0 : S_n > 0\}.$$

Prove that $E\tau = \infty$.

3) Take two numbers $\lambda, \mu > 0$ such that $\lambda + \mu < 1$. Show that the sequence $(1, \lambda + \mu, \lambda^2 + \mu^2, ...)$ cannot be the potential sequence of any renewal sequence.

4) (Problem 25.9 in the textbook) Decide for which $q \in (0, 1]$ the sequence $(1, 0, q, q, ...)$ is a potential sequence and find the corresponding waiting time distribution.