Theory of Probability and Measure Theory – Math 8652

Homework #8

1) Given a Markov chain on $S = \mathbb{Z}$ or $\mathbb{Z}^2$, let $u : S \to \mathbb{R}$ be a bounded harmonic function with respect to this Markov chain. Show that u is constant.

2) Consider a simple symmetric random walk on $\mathbb{Z}^d$, where $d = 1$ or 2. We know from Homework #7, Problem 3, that it is recurrent, i.e. the returning time is almost surely finite. Show that the random walk is null recurrent.

3) (Theorem 26.9 in the textbook) Let μ be a probability distribution on $\mathbb{Z}^+ = \{0, 1, 2, ...\}$ such that $\mu(\{2, 3, ...\}) > 0$. Let ρ be the probability generating function of μ. Consider a branching process having branching distribution μ. Show that the probability of extinction of the above process starting from 1 is the smallest root in [0,1] of the equation $c = \rho(c)$.

4) Fix $d \geq 2$ and let $S = \{1, 2, ..., d\}$. For $1 < i < d$, let $p(i, i \pm 1) = 1/2$ and let $p(1, 1) = p(d, d) = 1$. Find all invariant probability distributions.