

MATH 111 - Exam Two - Fall 2021

1. Let $p(x) = x^3 - 3x^2 + x + 2$.

1a. [2pts] Show that $p(2) = 0$.

1b. [10pts] Use (1a) to find all solutions to the equation $p(x) = 0$. (Give solutions in exact form).

2. [12pts] Sketch a graph of the parabola given by $y = 3x^2 - 9x + 2$. Include the coordinates (exact values) of the vertex and any intersections with the x-axis and the y-axis.

3. A 60 foot long ditch is to be dug. Alone, Kim can dig the ditch in 12 hours. Mike, if he works alone, can dig the ditch in 10 hours.

3a. [2pts] At what rate does Kim dig (in units of feet per hour)?

3b. [2pts] At what rate to Mike dig (in units of feet per hour)?

3c. [10pts] If Kim and Mike work together, how long does it take them to dig the entire ditch? (Give your answer rounded to the nearest minute).

4. [12pts] A recent study looked at “texting speed”, the rate (in words per minute) that people could write a text message. People in their teenage years could text at a rate 10 words per minute faster than those over forty years old.

Suppose it took those over forty years old 2 minutes longer to produce a 200 word text message than it did for teenagers. Let X represent the rate at which teenagers can text. Write an equation for the variable X using the information above. (Note: You don't need to solve the equation.)

5. Let $f(x) = \begin{cases} 1 - 2x & \text{if } x < 1 \\ x^2 & \text{if } x \geq 1 \end{cases}$

5a. [6pts] Sketch a graph of the function.

5b. [8pts] Determine the range of the function.

5c. [8pts] Determine the interval of values of y for which $f(x) = y$ has exactly one solution.

6. Let $f(x) = \frac{3x - 5}{x^2 + 3x - 10}$.

6a. [8pts] Create a sign chart to determine where $f(x)$ is positive and where it is negative.

6b. [8pts] Use the sign chart to produce a graph of $f(x)$. Make certain the graph shows appropriate asymptotic behavior (including end behavior).

7. [12pts] Suppose $f(x)$ is a function with domain $[1, 10]$ and range $[-3, 3]$. Let $g(x) = 1 - 2f(3x + 1)$.

Determine the domain and range of $g(x)$.