MATH 111 - Final Exam - Fall 2021

1. [10pts] Create a number line sign chart to determine where the following function is positive, negative, zero, and undefined. Then use that information to sketch a graph of the function. (Make sure your graph indicates where this graph crosses the x-axis and y-axis and has the appropriate end behavior).

$$f(x) = \frac{2x - 3}{x^2 - 3x - 4}$$

2. [8pts] If $\log_a(x) = 3$, $\log_a(y) = -2$, and $\log_a(z) = 5$ then what is the value of $\log_a(\frac{z^2\sqrt{x}}{y^3})$?

3. [10pts] Use the fact that $x = -2$ is a solution to the equation

$$x^3 + x^2 - 7x - 10 = 0$$

to find two other solutions. (Give your answers in exact form, not as decimal approximations).

4. [8pts] Find the center and radius of the circle given by the equation

$$x^2 - 7x + y^2 + 2y = 0$$

5. [10pts] The rental costs for a cargo van for one day are \$29.99 plus \$1.25 per mile driven. Let variable x represent the number of miles driven. Let variable y represent the total rental cost for renting the van and driving x miles. First find an equation relating x and y. Then answer the following question:

If the total rental cost was \$189.99, how far was the van driven?

6. [10pts] The function $f(x)$ has a domain of $[-3, 5]$ and a range of $[2, 9]$. Suppose $g(x) = \frac{1}{2}f(2x - 3) - 4$. Determine the domain and range of $g(x)$.

7. [10pts] Solve the following equation for the variable x. (Give an exact answer as well as a decimal approximation rounded to the nearest thousandth).

$$\frac{200}{5 + 15e^{-x/4}} = 20$$

8. [8pts] Sketch a graph of the following piecewise-defined function. Then determine the range of the function.

$$f(x) = \left\{ \begin{array}{cc} 3x - 8 & \text{if} \quad x \leq 3 \\ 5 - x & \text{if} \quad x > 3 \end{array} \right\}$$

9. [8pts] A manufacturing plant has two machines which produce bolts. The newer of the two machines can produce 25 more bolts per minute than can the older machine.

Let variable x represent the rate at which the newer machine produces bolts (in units of bolts per minute). If both machines are set to produce 5000 bolts, write down an expression for the span of time (in minutes) between when the newer machine finishes and when the older machine finishes.

(Note: There's no equation to solve here. You are only being asked to write an expression (in terms of x) for the quantity described above).

10. [8pts] Rewrite the following by converting it into a single rational expression:

$$\frac{6}{x^2 - 4x - 5} - \frac{2x - 3}{x^2 - 7x + 10}$$

11. [10pts] Carbon-14 is a radioactive isotope found in organic matter. It has a very long half-life (5730 years) and is useful for archeologists as they attempt to age artifacts at dig sites. The amount of the substance is modeled by the exponential decay equation $A(t) = Ce^{-rt}$. Suppose that an object should initially have had 125 grams of Carbon-14, but that it currently has 107 grams of that isotope. How old is the artifact? (Hint: To start this problem, you'll need to determine the values of C and r based on the information given).