

**MATH 112, FINAL EXAM, SPRING 2023**

INSTRUCTOR: TUAN PHAM

Name

**Instructions:**

- This is a closed-book exam, 2 hours long.
- A 4" x 6" handwritten single-sided note card is allowed. A scientific calculator is allowed. Graphing/programmable/transmittable calculators are not allowed.
- For Problems 1-8, fill in the bubbles on this front page. To each problem, only one answer is correct.
- For Problems 9-11, make sure to show all necessary steps. Mysterious answers will receive little or no credit.

1.     A    B    C    D
2.     A    B    C    D
3.     A    B    C    D
4.     A    B    C    D
5.     A    B    C    D
6.     A    B    C    D
7.     A    B    C    D
8.     A    B    C    D

Problem	Possible points	Earned points
1-8	16	
9	5	
10	6	
11	6	
Total	30	

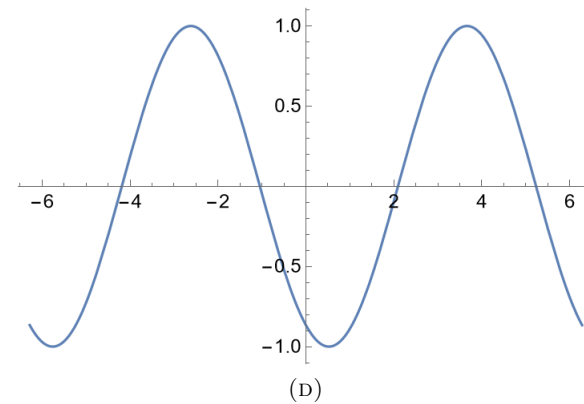
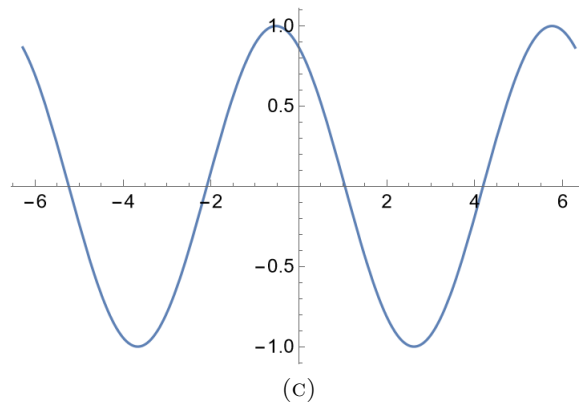
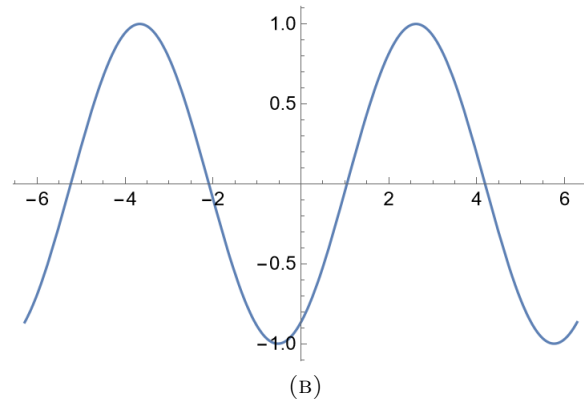
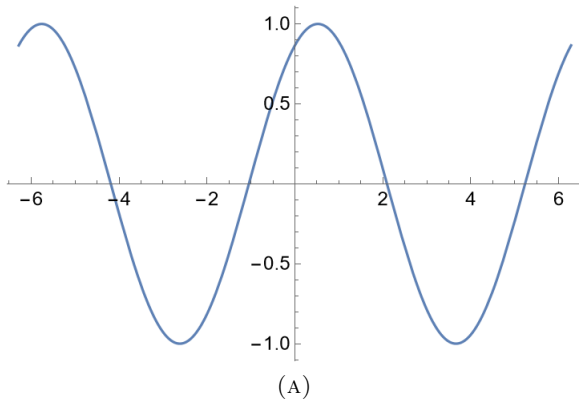
**Problem 1.** (2 points) Which of the following is the *range* of the function  $\arccos(x)$  ?

- A.  $[0, \pi/2]$
- B.  $[0, \pi]$
- C.  $[0, 2\pi]$
- D.  $[-\pi/2, \pi/2]$

**Problem 2.** (2 points) Choose the correct value of  $\sin\left(\arcsin\left(\frac{2}{\sqrt{2}}\right)\right)$

- A.  $\frac{\pi}{4}$
- B.  $\frac{\sqrt{2}}{2}$
- C.  $\frac{2}{\sqrt{2}}$
- D. undefined

**Problem 3.** (2 points) Choose the correct graph of the function  $f(x) = \sin\left(x - \frac{\pi}{3}\right)$ .



**Problem 4.** (2 points) Choose the expression that is equal to  $\cos(x - \frac{\pi}{4})$ .

- A.  $\frac{\sqrt{2}}{2}(\cos x + \sin x)$
- B.  $\frac{\sqrt{2}}{2}(\cos x - \sin x)$
- C.  $\frac{\sqrt{2}}{2}(-\cos x + \sin x)$
- D.  $\frac{\sqrt{2}}{2}(-\cos x - \sin x)$

**Problem 5.** (2 points) How many solutions  $x \in [0, 2\pi]$  does the equation  $\cos(2x) = 0$  have?

- A. 2
- B. 3
- C. 4
- D. 5

**Problem 6.** (2 points) A point has Cartesian coordinates  $(-\frac{1}{3}, \frac{\sqrt{3}}{3})$ . Choose the correct polar coordinates of this point.

- A.  $(6, \frac{\pi}{3})$
- B.  $(6, \frac{2\pi}{3})$
- C.  $(\frac{2}{3}, -\frac{2\pi}{3})$
- D.  $(-\frac{2}{3}, -\frac{\pi}{3})$

**Problem 7.** (2 points) Which of the following is equal to  $(\sqrt{3} + i)^4$  ?

- A.  $16 \operatorname{cis}(\frac{2\pi}{3})$
- B.  $16 \operatorname{cis}(\frac{4\pi}{3})$
- C.  $\frac{1}{16} \operatorname{cis}(\frac{2\pi}{3})$
- D.  $\frac{1}{16} \operatorname{cis}(\frac{4\pi}{3})$

**Problem 8.** (2 points) Consider three points  $A(0, 1)$ ,  $B(-2, 3)$ ,  $C(1, -1)$ . Find the length of the vector  $\overrightarrow{AB} + \overrightarrow{CA}$ .

- A.  $\sqrt{7}$
- B. 1
- C. 5
- D.  $\sqrt{17}$

**Problem 9.** (5 points) Find all  $x \in [0, 2\pi]$  satisfying the equation

$$\sin(2x) = -\cos x$$

**Problem 10.** (6 points) Use suitable trigonometric identities to prove that

$$\sin(3x) = 3 \sin x - 4 \sin^3 x$$

Make sure to state the name of each identity you use.

**Problem 11.** (6 points) Consider a triangle with vertices  $A$ ,  $B$ ,  $C$ . Suppose the angle at  $A$  is  $30^\circ$ , the angle at  $B$  is  $45^\circ$ , and the length of  $AB$  is 5. Find the missing angle and side lengths. Round your answers up to four digits after the decimal point.