MATH 251, FINAL EXAM, FALL 2022

INSTRUCTOR: TUAN PHAM

Name

Instructions:

- This is a closed-book exam, 90 minutes long.
- A single sided, handwritten, 3" x 5" note card is allowed. A scientific calculator is allowed. Graphing/programmable/transmittable calculators are not allowed.
- For Problems 1-16, fill in the bubbles on this front page. To each problem, only one answer is correct. Problems 9-16 are for bonus credit.
- For Problems 17, 18 and 19, make sure to show all necessary steps. Mysterious answers will receive little or no credit.

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Problem	Possible points	Earned points
1-8	16	
17	5	
18	5	
19	5	
Total	31	
Bonus (9-16)	8	

Problem 1. (2 points) The derivative of $\sqrt{x^2+x}$ is

A. $\sqrt{2x+1}$

B. $\frac{2x+1}{2\sqrt{x^2+x}}$

C. $\frac{1}{2\sqrt{x^2+x}}$

D. $\frac{1}{2\sqrt{2x+1}}$

Problem 2. (2 points) Let $f(x) = \sin\left(\frac{\pi}{x}\right)$. Which of the following is the correct value of $f'(1)$?

A. -1

B. π^2

C. $-\pi$

D. π

Problem 3. (2 points) Let x and y be related to each other through the equation $xy + y^2 = x$. Viewing y as a function of x, find y'.

A. $\frac{1-y}{x+2y}$

B. $\frac{-y}{x+2y}$

C. $\frac{1-3y}{x}$

D. $1-2y$

Problem 4. (2 points) The linearization of the function $f(x) = \sqrt{x}$ when $x \approx 1$ is

A. $f(x) \approx 1$

B. $f(x) \approx \frac{1}{2}x + \frac{1}{2}$

C. $f(x) \approx x$

D. $f(x) \approx 1 + \frac{1}{2\sqrt{x}}(x-1)$

Problem 5. (2 points) All the critical numbers of $f(x) = x^3 + x^2 - x$ are

A. 0 and $\frac{-1\pm\sqrt{5}}{2}$

B. $-\frac{1}{3}$

C. -1 and $\frac{1}{3}$

D. 1 and 3

Problem 6. (2 points) Let f be a function that is continuous on $[-1, 1]$, differentiable on $(-1, 1)$, and $f(-1) = 3$, $f(1) = -1$. Which of the following statement is correct?

A. There exists a number $c \in (-1, 3)$ such that $f'(c) = -2$.

B. There exists a number $c \in (-1, 3)$ such that $f'(c) = 2$.

C. There exists a number $c \in (-1, 1)$ such that $f'(c) = -2$.

D. There exists a number $c \in (-1, 1)$ such that $f'(c) = 0$.

Problem 7. (2 points) Let f be a function that is continuous on $[-1, 1]$, differentiable on $(-1, 1)$, and $f(-1) = 3$, $f(1) = -1$. Which of the following results says that the equation $f(x) = 0$ has at least one root?

A. Fermat's Lemma

B. Rolle's Theorem

C. Mean Value Theorem

D. Intermediate Value Theorem

Problem 8. (2 points) Let f be a differentiable function such that f' is continuous and $f'(x) < 0$ for $x < 1$ and $f'(x) > 0$ for $x > 1$. Which of the following statement is correct?

A. f attains a local minimum at $x = 1$.

B. f attains a local maximum at $x = 1$.

C. $x = 1$ is an inflection point of f.

D. The equation $f(x) = 1$ has at least one root.

Problem 9. (1 point) Find all the horizontal asymptotes of $f(x) = \frac{x}{x^2-2x+1}$.

A. $y = 0$

B. $x = 1$

C. $y = 0$ and $x = 1$

D. No horizontal asymptotes

Problem 10. (1 point) Choose the correct value of

$$\lim_{x\to-\infty} \frac{\sqrt{x^2+1}}{x}$$

A. 1

B. -1

C. 0

D. $-\infty$

Problem 11. (1 point) Let

$$f(x) = \begin{cases} x+1 & \text{if} \quad x < -1, \\ cx-1 & \text{if} \quad x \geq -1. \end{cases}$$

For which value of c is f a continuous function?

A. 1

B. 0

C. -1

D. -2

Problem 12. (1 point) The derivative of $f(x) = x \sin\left(\frac{1}{x}\right)$ is

A. $\cos\left(\frac{1}{x}\right)$

B. $\cos\left(-\frac{1}{x^2}\right)$

C. $-\frac{1}{x^2}\cos\left(\frac{1}{x}\right)$

D. $\sin\left(\frac{1}{x}\right) - \frac{1}{x^2}\cos\left(\frac{1}{x}\right)$

Problem 13. (1 point) The tangent line to the parabola $y = x^2$ at the point $(1, 1)$ is

A. $y = x$

B. $y = 2x^2 - 2x + 1$

C. $y = 2x - 3$

D. $y = 2x - 1$

Problem 14. (1 point) The tangent line to the unit circle $x^2 + y^2 = 1$ at the point $\left(\frac{3}{5}, \frac{4}{5}\right)$ is

A. $\frac{3}{4}$

B. $-\frac{3}{4}$

C. $\frac{4}{3}$

D. $-\frac{4}{3}$

Problem 15. (1 point) Suppose $f'(1) = 0$. Choose the correct statement.

A. $x = 1$ is a critical number of f.

B. $x = 1$ is an x-intercept of f.

C. $x = 1$ is an inflection point of f.

D. $x = 1$ is a vertical asymptote of f.

Problem 16. (1 point) On the interval $[-2, -1]$, the graph of the function $f(x) = x^3 - 4x$ is

A. increasing

B. decreasing

C. concave downward

D. concave upward

Problem 17. (5 point) Let $f(x) = x^3 - 3x$.

(a) Find all the critical numbers of f.
(b) Draw a fluctuation chart of f. Indicate in that chart the local minimum and local maximum.
(c) Find the inflection point of f.
(d) Sketch the graph of f.
(e) Find the minimum and maximum value of f when $x \in [-1.7, 1.7]$

Problem 18. (5 points) Use Newton's method to evaluate $\sqrt{2}$ with allowable error 0.0001. Make sure to write down the recursion formula before plugging in numbers.

Problem 19. (5 points) Show that the function $f(x) = 2x + \sin x$ has exactly one real root.