MATH 251 - Fall 2020 - Exam One

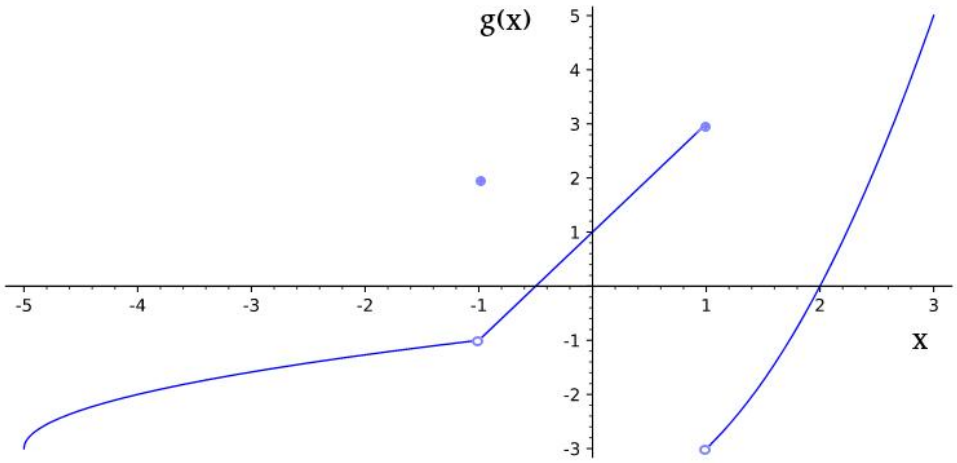
1. Determine the exact values of the limits below. Show all relevant algebraic work.

1a. [8pts] $\lim_{x \to 2} \dfrac{x^2 + 3x - 10}{x^2 + x - 6}$

1b. [8pts] $\lim_{x \to 2} \dfrac{6 - 3x}{\sqrt{2x} - 2}$

1c. [8pts] $\lim_{x \to 0} \dfrac{\tan(x)}{x^2 - 7x}$

2. Let $g(x)$ be described by the graph below. Determine the values indicated below the graph or indicate that they do not exist.



2a. [8pts] $g(-1)$

2b. [8pts] $\lim_{x \to -1} g(x)$

2c. [8pts] $g(1)$

2d. [8pts] $\lim_{x \to 1} g(x)$

3. Let $f(x) = \sqrt{x}$ and answer the questions below.

3a. [9pts] Write an expression (in terms of a limit) for the slope of the tangent line to the graph of $f(x)$ at the point where $x = 9$.

3b. [9pts] Calculate $f'(9)$ by evaluating the limit you wrote in (3a). (You may wish to check your work by differentiating using the power rule as well, but this question is specifically asking you to evaluate the limit).

4. [8pts] Let $g(x) = \left\{ \begin{array}{ll} x^2 + 2x & \text{if } x \leq -2 \\ 3x + C & \text{if } x > -2 \end{array} \right\}$

Determine the value of C that makes $g(x)$ a continuous function.

5. A radioactive substance is decaying, so that the amount of the substance is decreasing over time. The amount of substance remaining is given by a function $Q(t)$ where t is time in hours after measurements of the substance have begun and $Q(t)$ is mass of the substance (in grams) at that time. Answer the questions below.

5a. [6pts] Write a sentence describing the meaning of $Q(10) = 34$. Be sure to include the units attached to both numbers as part of your description.

5b. [6pts] Write a sentence describing the meaning of $Q'(10) = -3.4$. Be sure to include the units attached to both numbers as part of your description.

5c. [6pts] Radioactive substances have a "half life" which is a fixed amount of time it takes for the mass of such a substance to be reduced to half. (That is, it takes just as much time for a substance to decay from 100 grams to 50 grams as it does for it to decay from 50 grams to 25 grams). Given that fact, if $Q'(10) = -3.4$ would you expect $Q'(20) = -3.4$ as well? Or would you expect $Q'(20) < -3.4$? Or $Q'(20) > -3.4$? Pick one and explain.