MATH 251 - Exam Two

1. An engineer is testing a leaky pipe and measuing the amount of water that is being drained from the pipe due to the leak. He starts his measurements at time $t = 0$ hours and finds, for the next ten hours, that the total amount of leaked water at time t is given by the equation

$$f(t) = 0.06t^2 + 10t$$

where $f(t)$ is measured in milliliters.

1a. [8pts] At what rate (in milliliters / hour) is the pipe leaking at time $t = 0$?

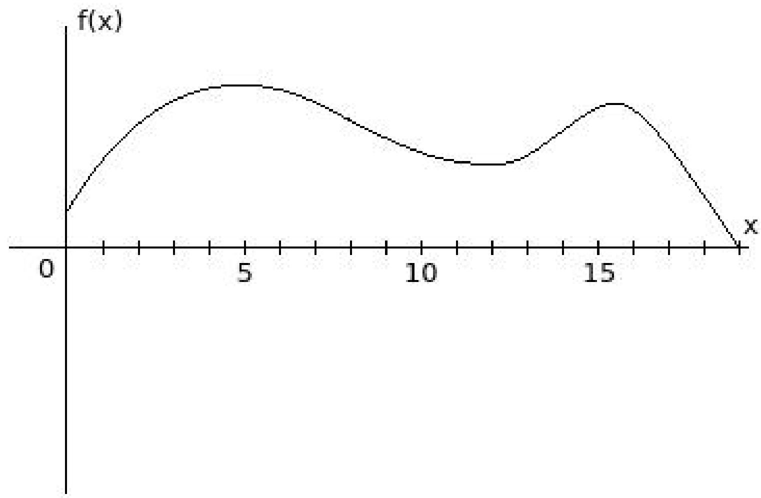
1b. [8pts] At what rate is the pipe leaking at time $t = 10$?

2. [10pts] Consider the curve described by the equation $x \cos y = x^2 + y^2$. Use implicit differentiation to find an expression for $\frac{dy}{dx}$ in terms of x and y. (You don't need to simply this expression, but you should have the equation solved for dy/dx).

3. [15pts] An intersection between two roads is located 15 miles due west of a cell phone tower. At time $t = 0$ a woman is driving north from the intersection at a constant rate of 40 miles per hour. At what rate is the distance between the woman and the cell phone tower increasing 30 minutes later?

4. [12pts] Find the absolute minimum and maximum values of the function $g(x) = 12 + 4x - x^2$ on the interval [0,5].

5. Consider the graph below of the function $f(x)$. Answer the questions below the graph.



5a. [8pts] On what interval(s) is $f'(x) > 0$?

5b. [8pts] On what interval(s) is $f''(x) > 0$?

6. [11pts] Suppose that $f(x)$ and $g(x)$ are "inverse functions" which means that $f(g(x)) = x$ for every value of x. Suppose that $f(x)$ is a function that is its own derivative. That is, $f'(x) = f(x)$. Differentiate both sides of the equation $f(g(x)) = x$ and use the result to find an expression for $g'(x)$.

7. [20pts] Use curve sketching techniques to sketch a graph of the function $f(x) = 2\sqrt{x} - x$ as accurately as possible. Be sure to find all x-intercepts and y-intercepts as well as all local maxima or minima and inflection points. Also be sure to include end behavior as $x \to \infty$.