MATH 251 - Final Exam

1. [8pts] Sunlight is shining down on a pond. The intensity of light below the surface drops off rapidly as a function of depth according to a function $L(d)$ where L is the intensity in units of lux and d is distance in units of meters.

Describe, in your own words, the physical meaning of the equation

$$L'(5) = -0.2.$$

(A complete explanation should indicate what units should be attached to the number -0.2).

2. [8pts] Find the general anti-derivative $f(x)$ of the function

$$f'(x) = \sin(2x) + x^2.$$

3. [12pts] Calculate the following limit:

$$\lim_{h\to 0} \frac{\dfrac{1}{(x+h)^2} - \dfrac{1}{x^2}}{h}$$

4. [8pts] Find a real number c so that the following function is continuous.

$$f(x) = \left\{ \begin{array}{ll} x^2 - x & \text{if } x < -1 \\ 5x + c & \text{if } x \geq -1 \end{array} \right\}$$

5. [12pts] Find, exactly, the largest and smallest values assumed by the function $f(x) = x^3 - 5x^2 + 3x + 1$ on the domain $[0, 2]$.

6. [10pts] Sketch a graph of the region whose area is described by the integral $\int_0^4 4x - x^2\,dx$. Then use the Fundamental Theorem of Calculus to calculate that area.

7. [12pts] Since the Earth rotates once every 24 hours, the rays of the setting sun strike the ground at an angle that is decreasing at a rate of $\frac{360 \text{ degrees}}{24 \text{ hours}}$. How fast is the length of the shadow of a 40 foot high flagpole increasing when the angle of the sun's rays strike ground at a 30^o angle? (Draw a picutre of the situation and then solve the problem).

8. [12pts] Use curve sketching techniques to sketch the graph of the function

$$f(x) = \frac{1}{x^2 + 1}.$$

Be sure to include the exact values of any maxima, minima, and inflection points, together with any asymptotic behavior.

9. [8pts] It is a fact (which you'll learn about if you take Calc II) that the function $f(x) = e^x$ is its own derivative. That is, $\frac{d}{dx}(e^x) = e^x$. Use this fact to answer the question below:

Let $h(x)$ be some differentiable function and let $g(x) = e^{h(2x)}$. Find an expression for $g'(x)$.

10. [10pts] Let $f(x) = x^3 - 5x + 1$. Note that $f(0) = 1$ while $f(1) = -3$, which means that $f(x)$ must be zero for some x in the interval $(0, 1)$. Use $x_1 = 0$ as your initial guess and apply two iterations of Newton's method to estimate the value of the zero of this function.