MATH 252, FINAL EXAM, WINTER 2023

INSTRUCTOR: TUAN PHAM

Name

Instructions:

- This is a closed-book exam, 2 hours long.
- A 4" x 6" handwritten single-sided note card is allowed. The Table of Integral in References 6-10 in the back of the textbook is allowed. A scientific calculator is allowed. Graphing/programmable/transmittable calculators are not allowed.
- For Problems 1-12, fill in the bubbles on this front page. To each problem, only one answer is correct.
- For Problems 13, 14 and 15, make sure to show all necessary steps. Mysterious answers will receive little or no credit.

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Problem	Possible points	Earned points
1-10	20	
11-12 (extra credit)	4	
13	5	
14	5	
15	5	
Total	35	

Problem 1. (2 points) Choose the correct value of the limit

$$\lim_{x \to \infty} \frac{1 - e^{-2x}}{1 + e^{-x}}$$

A.   0

B.   1

C. -2

D. ∞

Problem 2. (2 points) Choose the correct derivative of the function $f(x) = \sqrt{1 + 2^x}$.

A. $\dfrac{\ln 2}{2} \dfrac{2^x}{\sqrt{1 + 2^x}}$

B. $\dfrac{1}{2 \ln 2} \dfrac{2^x}{\sqrt{1 + 2^x}}$

C. $\sqrt{2^x (\ln 2)}$

D. $\dfrac{1}{2} \dfrac{1}{\sqrt{2^x (\ln 2)}}$

Problem 3. (2 points) Choose the correct derivative of the function $f(x) = \arctan(x^2)$.

A. $\frac{1}{x^4 + 1}$

B. $\frac{2x}{x^2 + 1}$

C. $\frac{1}{(x^2 + 1)^2}$

D. $\frac{2x}{x^4 + 1}$

Problem 4. (2 points) The domain of the function $f(x) = \tanh(x)$ is

A. $(-\infty, \infty)$

B. $(-1, 1)$

C. $(0, \pi)$

D. $(0, \pi/2)$

Problem 5. (2 points) Evaluate the limit
$$\lim_{x \to 1} \frac{x^3 - 3x + 2}{x^3 - x^2 - x + 1}$$

A. 0

B. 1

C. 3/2

D. Does not exist

Problem 6. (2 points) Evaluate the integral
$$\int_0^\pi \sin(x)\cos^2(x)dx$$

A. $-2/3$

B. $2/3$

C. 4

D. $\pi^3/3$

Problem 7. (2 points) The area of the region under the curve $y = \ln x$ and above the x-axis, between $x = 1$ and $x = 2$, is

A. $2\ln(2) - 2$

B. $2\ln(2) - 3$

C. $2\ln(2) + 1$

D. $2\ln(2) - 1$

Problem 8. (2 points) Find the value of
$$\int_1^\infty \frac{1}{x^2}dx$$

A. 2

B. -1

C. 1

D. Does not exist

Problem 9. (2 points) Choose the correct derivative of $f(x) = x^x$.

A. 1

B. x^x

C. $(\ln x)x^x$

D. $(1 + \ln x)x^x$

Problem 10. (2 points) The function $f(x) = e^{-x+3\ln x}$ has an equivalent form as

A. $x^3 e^{-x}$

B. $3xe^{-x}$

C. e^{2x}

D. $e^{-x}3^x$

Problem 11. (2 points) Choose the correct antiderivative of the function $f(x) = \frac{1}{9+4x^2}$.

A. $\frac{1}{3}\arctan\left(\frac{2x}{3}\right) + C$

B. $\frac{1}{6}\arctan\left(\frac{2x}{3}\right) + C$

C. $\frac{1}{4}\arctan\left(\frac{x}{3}\right) + C$

D. $\frac{1}{6}\arctan\left(\frac{x}{3}\right) + C$

Problem 12. (2 points) Let E be the solid obtained by rotating the region under the curve $y = 2x$, $0 \le x \le 1$, about the x-axis. The volume of E is

A. π

B. $2\pi/3$

C. $4\pi/3$

D. $\pi/3$

Problem 13. (5 points) Use the method of substitution or integration by parts to evaluate

$$\int_0^\pi x\sin(2x)dx$$

Problem 14. (5 points) Evaluate the integral

$$\int_2^3 \frac{x}{x^2 + x - 2}dx$$

Problem 15. (5 points) Evaluate the area under the curve $y = 4 - x^2$ and above the line $y = 3$.