# MATH 252, MIDTERM, WINTER 2023

INSTRUCTOR: TUAN PHAM

| Name |
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**Instructions:**

- This is a closed-book exam, 50 minutes long.
- A 4" x 6" handwritten single-sided note card is allowed. A scientific calculator is allowed. Graphing/ programmable/ transmittable calculators are not allowed.
- For Problems 1-7, fill in the bubbles on this front page. To each problem, only one answer is correct.
- For Problems 8, 9 and 10, make sure to show all necessary steps. Mysterious answers will receive little or no credit.

1. Ⓐ Ⓑ Ⓒ Ⓓ
2. Ⓐ Ⓑ Ⓒ Ⓓ
3. Ⓐ Ⓑ Ⓒ Ⓓ
4. Ⓐ Ⓑ Ⓒ Ⓓ
5. Ⓐ Ⓑ Ⓒ Ⓓ
6. Ⓐ Ⓑ Ⓒ Ⓓ
7. Ⓐ Ⓑ Ⓒ Ⓓ

| Problem | Possible points | Earned points |
| --- | --- | --- |
| 1-7 | 14 | |
| 8 | 5 | |
| 9 | 5 | |
| 10 | 5 | |
| Total | 29 | |

**Problem 1.** (2 points) Choose the correct antiderivative of the function $f(x) = x - \sin(2x)$.

A. $1 - 2\cos(2x)$

B. $\frac{x^2}{2} - \frac{\cos(2x)}{2}$

C. $\frac{x^2}{2} + \frac{\cos(2x)}{2} + 1$

D. $1 + 2\cos(2x)$

**Problem 2.** (2 points) Choose the correct sigma notation for the sum

$$1 + \frac{2}{3} + \frac{3}{3^2} + \frac{4}{3^3} + \frac{5}{3^4} + ... + \frac{100}{3^{99}}$$

A. $\sum_{k=1}^{99} \frac{k+1}{3^k}$

B. $\sum_{k=1}^{100} \frac{k}{3^{k-1}}$

C. $\sum_{k=0}^{100} \frac{1}{3^k}$

D. $\sum_{k=1}^{100} \frac{3^k-1}{3^k}$

**Problem 3.** (2 points) The function

$$f(x) = \frac{2x-1}{(-x^2+x)^2}$$

has an antiderivative

$$F(x) = \frac{1}{-x^2+x}$$

What is the area under the curve $y = f(x)$ where $2 \leq x \leq 3$ ?

A. 11/18

B. 11/12

C. 1/3

D. 2/3

**Problem 4.** (2 points) The limit

$$\lim_{n\to\infty} \sum_{k=1}^{n} \left(\frac{k}{n}\right)^2 \frac{1}{n}$$

represents which of the following integrals?

A. $\int_0^1 x^2 dx$

B. $\int_0^1 x^3 dx$

C. $\int_0^1 \frac{1}{x^2} dx$

D. $\int_0^2 \frac{1}{x^2} dx$

**Problem 5.** (2 points) Evaluate the integral $\int_0^3 |x-1|dx$

A. 7/2

B. 9/2

C. 3/2

D. 5/2

**Problem 6.** (2 points) Find the inverse of the function $f(x) = \frac{x^2-1}{x^2+1}$ when $x > 0$.

A. $-\sqrt{\frac{1-y}{1+y}}$

B. $\sqrt{\frac{1-y}{1+y}}$

C. $-\sqrt{\frac{1+y}{1-y}}$

D. $\sqrt{\frac{1+y}{1-y}}$

**Problem 7.** (2 points) Choose the correct value of the limit

$$\lim_{x\to\infty} \left[\ln(2x^{10}+x^9) - \ln(x^{10})\right]$$

A. $\ln 2$

B. 3/4

C. 1

D. DNE

**Problem 8.** (5 points) Use the right-point Riemann sum with $n = 4$ to estimate the area under the curve $y = \ln x$ where $1 \leq x \leq 2$. Round your result to four decimal places.

**Problem 9.** (5 points) Find

$$\int x \sin(x^2 + 1) dx$$

using the substitution $u = x^2 + 1$.

**Problem 10.** (5 points) Evaluate

$$\int_0^1 x\sqrt{2-x}\,dx$$