Maple Lab 2

Recall the following basic commands in Maple:

- To declare a function $f(x) = \sin(\pi x^2)$:

```
f := x->sin(Pi*x^2)
```

- To graph a function $f(x)$ on the interval $[a, b]$:

```
plot(f(x),x=a..b)
```

- To graph two functions $f(x)$ and $g(x)$ on the interval $[a, b]$ on the same plot:

```
plot([f(x),g(x)],x=a..b)
```

- To compute the limits $\lim_{x\to a} f(x)$, $\lim_{x\to a^-} f(x)$, $\lim_{x\to a^+} f(x)$:

```
limit(f(x),x=a)
limit(f(x),x=a,left)
limit(f(x),x=a,right)
```

1 Practice

Compute the following limits using the `limit` command. Try to justify the result by graphing the function. Can you also justify it using limit laws?

1.
$$\lim_{x\to 0} \frac{\sqrt{3+x} - \sqrt{3}}{x}$$

2.
$$\lim_{x\to 6^-} \frac{2x+12}{|x+6|}$$

3.
$$\lim_{x\to -2} \frac{2-|x|}{2+x}$$

We write

$$\lim_{x\to a} f(x) = \infty$$

if the value of $f(x)$ can be made arbitrarily large by choosing x sufficiently close to a (on either side of a). Note that this implies that the limit doesn't exist. However, we still say that *the limit of $f(x)$ as x approaches a is equal to infinity*, which is more informative than just saying the limit of $f(x)$ doesn't exist. The notations

$$\lim_{x\to a^-} f(x) = \infty,\ \lim_{x\to a^+} f(x) = \infty,\ \lim_{x\to a} f(x) = -\infty,\ \lim_{x\to a^-} f(x) = -\infty,\ \lim_{x\to a^+} f(x) = -\infty$$

are similarly understood. If one of these six scenarios happens, then the vertical line $x = a$ is called a *vertical asymptote* of $f(x)$.

Graph the following functions and identify all vertical asymptotes.

4.
$$\ln(x^2 - 1)$$

5.

$$\frac{x^2+1}{2x^4+7x^3+7x^2+2x}$$

Use the command

```
factor(2*x^4+7*x^3+7*x^2+2*x)
```

to confirm your observation in Problem 5.

We write

$$\lim_{x\to\infty} f(x) = L$$

if the value of $f(x)$ can be made arbitrarily close to L by choosing x sufficiently large. The notation

$$\lim_{x\to-\infty} f(x) = L$$

is similarly understood. If one of these two scenarios happens, then the horizontal line $y = L$ is called a *horizontal asymptote* of $f(x)$. Try the command

$$\text{limit}(\sqrt{x+1} - \sqrt{x},\ x = \text{infinity})$$

6. To each of the following function, identify all horizontal asymptotes. Graph each function together with all of its horizontal asymptotes on the same plot.

$$\frac{\sqrt{x^2+1}}{x+1},\ \arctan(x),\ \frac{x}{e^x},\ \frac{x^5}{e^x},\ \frac{x^{10}}{e^x}.$$

   How fast does the exponential function grow, as x goes to infinity, compared to a polynomial?

7. Try this procedure in Maple:
   `compare:=proc(f,g)` (then Shift+Enter)
   `limit(f/g,x=infinity);` (then Shift+Enter)
   `end` (then Enter)
   Now type `compare(x,sqrt(x^2+1))`

2 To turn in

Turn in practice problems 1, 5, 6. Don't forget to justify your observation in Problem 5 using the command `factor` as explained.